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Lipid production optimization and optimal control of heterotrophic microalgae fed-batch bioreactor

Javad Abdollahi, Stevan Dubljevic *

Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4

HIGHLIGHTS

- Fed-batch heterotrophic microalgae growth and lipid production model is investigated.
- Nonlinear constrained programming is implemented to maximize the lipid concentration.
- Framework provided for lipid maximization in fed-batch microalgae bioreactor.
- Significant improvement in lipid and biomass concentration is achieved.

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ABSTRACT

Interior point optimization and model predictive control along with moving horizon estimator are used to maximize and regulate lipid production in a fed-batch heterotrophic microalgae cultivation of *Auxenochlorella protothecoides*. Motivation for microalgae bioreactor optimal control arises from the need to increase lipid production rate, which results in economic feasibility of microalgae bio-fuel production process. A complex time-varying microalgae fed-batch growth and lipid production model (De la Hoz Siegler et al., 2011) is used and a large-scale nonlinear programming optimization along with moving horizon estimator and model predictive control are applied to maximize the lipid concentration in the bioreactor. An optimal feeding strategy for lipid production is determined using the state-of-the-art interior point optimizer (IPOPT) solver. Moving horizon estimator (MHE) and model predictive controller (MPC) are used to estimate unmeasurable state (nitrogen concentration) and provide regulation of a highly nonlinear and time-varying microalgae growth process as a realizable real-time control strategy. In addition to the constrained large-scale optimization, naturally present input constraints (lower and upper bound on feed rates) and state constraints (lower bound on all concentration related states and upper bound on glucose concentration) are accounted for in an explicit manner with moving horizon estimator and model predictive controller. The estimator and controller design is based on a set of linearized models in microalgae growth fed-batch process. The paper provides a reliable and computationally efficient optimization, estimation and regulation procedure suitable for the real-time microalgae bioreactor operation. The procedure takes into account present constraints on inputs and states, and measurement noises present in the realistic operation conditions and is computationally efficient, along with the improvement in results with respect to previous methods.

1. Introduction

Economic and population growth have steadily increased the global energy demand. It is expected that the world will need almost 60% more energy, if current policies for energy management are held (Patil et al., 2008). Due to finite resources of fossil fuels and the impact of their consumption on environment, they do not make sustainable and reliable sources of energy any longer (Chisti, 2007; Hoffert et al., 2002; Khan et al., 2009). Therefore, there is a need for development of new energy sources such as biofuels that are renewable and environmentally sustainable. Currently, biodiesel and bio-ethanol are produced on a large industrial scale. They are replacements for petroleum for internal combustion engines, and are derived from food crops, which results in food–fuel conflict (Patil et al., 2008). On the other hand, biomass resources in many cases require appropriate compensation (e.g., replanting), otherwise their use may give rise to a massive biomass deficit and serious environmental problems (e.g., deforestation) (Patil et al., 2008).

In order to deal with the mentioned issues, one solution is to use heterotrophic microalgae as a substitute for the crops and other feedstock and consequently the produced biofuel can be used instead

* Corresponding author. Tel.: +1 780 248 1596; fax: +1 780 492 2881.
E-mail address: Stevan.Dubljevic@ualberta.ca (S. Dubljevic).

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of traditional fuels. Heterotrophic microalgae cultivation requires less water and land compared to phototrophic microalgae, crops and other feedstock, it is productive throughout the year and is characterized by a high growth rate (Hoffert et al., 2002). However, biofuel production from microalgae is not commercially viable due to low performance of bioreactors.

In order to achieve good performance and efficiency, bioreactors require advanced regulation procedures to ensure the performance and efficiency of bioprocesses. In general, a biological process is a network of complex biochemical reactions manipulated by enzymes (Mailleret et al., 2004; Farza et al., 1997). In fact, such kinetic networks give rise to highly complex and nonlinear dynamics of enzymes, nutrients and product concentrations in bioreactors. Modelling these dynamics with few states and parameters is a challenging task which may provide a relevant reduced order model of the process with usually time-varying and uncertain parameters. In addition to model uncertainties arising from complex biosystem dynamics, accurate and fast biochemical sensors are limited and difficult to realize in practice, which makes optimal bioreactor operation a challenging task. Therefore, in order to cope with such a complex biosystem, it is necessary to obtain sufficiently accurate and reliable information about the state and system parameters to come to efficient monitoring and regulation (Farza et al., 1997).

State and parameter estimation methods are well developed techniques in the process system science whose purpose is to handle unknown states and uncertainties in a model. There are several studies confirming that parameter and state estimation using linear and extended Kalman filter (EKF) fails or shows weak performance for the biological systems (Bastin and Dochain, 1990; Gonzalez et al., 2001; Selisteanu et al., 2007). Hence, using nonlinear observers for the state and parameter estimation of bioreactors has been successful and attracted much attention in recent years (Alcaraz-Gonzlez et al., 2005; Farza et al., 1998). In the work of Alcaraz-Gonzlez et al. (2005), the state estimation scheme is designed based on an asymptotic observer with a tuneable convergence rate. This robust observer is verified numerically by simulation on the wastewater treatment model. Farza et al. (1998) studied kinetic rates estimation in bioreactors and provided a theoretical framework for the kinetic rate estimation in bioreactors. Along the same line, Zhang and Guay (2002) used an adaptive nonlinear observer to estimate states and parameters of a microbial growth model. They used Lyapunov stability techniques and verified their approach by numerical simulations.

On the other hand, with respect to the control synthesis, predictive control methodologies can be used with state estimation to solve the control problem while handling the governing constraints. Sendrescu et al. (2011), El Bahja et al. (2009) and Tebbani et al. (2010) used nonlinear model predictive control (NMPC) for state regulation. Sendrescu et al. (2011) used NMPC to regulate sates in a nonlinear bioprocess with known states and with no constraints present. Nonlinear model predictive control is used for unconstrained state regulation in a lipase production bioprocess, while states are estimated using the Kalman filter (El Bahja et al., 2009). Tebbani et al. (2010) converted a constrained bioprocess control problem to an unconstrained nonlinear programming, and solved the constrained control problem to maximize cultures of *Escherichia coli*.

One of the strategies to ensure that the efficiency of lipid production is guaranteed is to force the microalgae states to track a reliable predefined reference trajectory. Using the growth model, the states and inputs evolution can be determined such that the maximum lipid density is achieved. Calculation of the optimal trajectory is a difficult task due to complexity of the model, constrained states and inputs and also due to the time scale of the optimization problem. For example, the lipid production optimization problem has been handled empirically by Hsieh and Wu (2009) and Doucha and Lvansky (2012) to find a strategy to maximize cell density based on the experimental results. Wu and Shi (2007) optimized the cell density using a hybrid neural network model.

There are three main approaches for nonlinear constrained optimization. Sequential quadratic programming (SQP), Interior point and nested projection methods (Biegler, 2010). SQP algorithms are based on the extension of Newton's method to quadratic problems and the computational expense of solution is limited to the quadratic problems solution. Active set selection and linear algebra used to solve KKT conditions determine the computational efficiency of the algorithm. These algorithms use few functions and derivative evaluations, but in the case of large-scale programs and large number of constraints they are computationally expensive due to necessity to choose an active set. Interior point or barrier methods are also based on Newton's method but they do not use the active set and they deal with relaxed KKT conditions; therefore, they can be used to handle large-scale programs. Nested projection methods are more efficient in the case of highly nonlinear objective function and constraints. They do not solve KKT conditions simultaneously, but decompose the problem and then use Newton's type methods, and as consequence they require more function evaluations (Biegler, 2010).

The Interior Point Optimizer (IPOPT) developed by Wachter and Biegler (2006), is a large-scale optimization solver for constrained nonlinear programs and can be applied to the lipid production model. The solution provided by the IPOPT might not achieve global optimum solution. Theoretically, the solution for the IPOPT solver is not global, and there is also a proven example (Wachter and Biegler, 2000) that the interior point method fails to globally converge. However, practically, searching within an active set enforced by constraints, the possibility to achieve a global solution is more likely. In addition, multiple runs with different initial guesses over the search domain is helpful to avoid local optimums. IPOPT is used to optimize bioprocesses, for example Estrada et al. (2009) used IPOPT to solve a large-scale nonlinear program of algae growth in the water reservoir, and IPOPT is also used for optimal management of the water treatment bioreactor by Alvarez-Vzquez et al. (2010).

In this work, microalgae growth and lipid production model are used in a large-scale nonlinear programming format to obtain optimal feeding strategy and consequent state trajectories in the presence of physically relevant constraints. The large-scale constrained optimization solution realized by IPOPT uses perfect model and yields desired optimal input evolutions. However, the optimization cannot account for the influence of possible disturbances in the feeds or bioreactor conditions, so that the obtained reference trajectory provides a desired state evolution to form a sequence of locally reduced and linearized models to synthesize the control and estimation strategy for the regulation of the lipid production rate. Subsequent linear time invariant models are used by the constrained moving horizon estimator (MHE) to construct full states of the process, since some of the states are not available for direct measurements (nitrogen concentration). With the real-time knowledge of the states, model predictive reference trajectory tracking is implemented to maintain the optimal performance of the process.

This work is based on the experimental work of De la Hoz Siegler et al. (2012). The contribution of the present work focuses on the predetermined optimal trajectory, simplicity for scaled-up implementation and also on handling constraints in estimation, control and optimization procedure. Tracking the predetermined optimal reference trajectory is a reliable way to maximize the lipid concentration in a bioreactor, which maintains the efficiency of a single bioreactor at a desired level, and can be applied to

different bioreactors to achieve the same properties of lipid. Using linear models for control and estimation guarantees the convexity of optimization in model predictive control, and therefore reduces the computational effort. In addition, linear models employed in algorithm simplify the algorithm implementation and realization on embedded controllers. Moreover, model predictive techniques used for control and estimation can handle constraints on inputs and states in an explicit manner which is not common in conventional control algorithms.

In the ensuing sections we provide a description of the microalgae bioreactor model, with the microalgae mathematical model. The following three sections deal with feeding strategy optimization, formulation of moving horizon estimator (MHE) for the state estimation, and model predictive controller (MPC) for the reference trajectory tracking. Finally, we provide simulation results and biological interpretation of the results with the emphasis on the possible presence of noise in the process regulation.

2. Microalgae growth and lipid production model

Microalgae growth takes place in a bioreactor vessel where microalgae cells grow in the presence of required nutrients and essential substances. Depending on the feeding strategy, different pathways of the growth and lipid production can be achieved by microalgae species (De la Hoz Siegler et al., 2011). These growth and lipid production pathways are governed by biochemical reactions in the microalgae cells. Detailed modelling of all these reactions is not a feasible task at the moment. The most common approach to derive a mathematical model is to use a few most effective variables in the process. The mathematical model is based on the material balance applied to the bioreactor and is given in Eqs. (1)–(6). The model parameters are identified using the experimental data in De la Hoz Siegler et al. (2011):

$$\frac{dS_1}{dt} = -\rho x + s_1^i \frac{f_1^i}{V} - S_1 D \tag{1}$$

$$\frac{dS_2}{dt} = -\frac{1}{Y_{x/s}}\mu x - \frac{1}{Y_{p/s}}\pi x - k_m x + s_2^i \frac{f_2^i}{V} - S_2 D \tag{2}$$

$$\frac{dx}{dt} = \mu x - xD \tag{3}$$

$$\frac{dp}{dt} = \pi x - \frac{1}{Y_{x/p}}\mu x - pD \tag{4}$$

$$\frac{dq}{dt} = \rho x - \frac{1}{Y_{x/q}}\mu x - qD \tag{5}$$

$$\frac{dV}{dt} = VD - f^o \tag{6}$$

This model, given by Eqs. (1)–(6), represents the heterotrophic growth and lipid production of *Auxenochlorella protothecoides*. The model is based on the assumption that algae cells are composed of three components, active biomass (x), lipid content (p), and nitrogen content (q). These three components can transfer to each other with constant rates ($Y_{i/j}$). Another important variable is glucose concentration (S_2), which is used to support the cell growth and lipid production, while nitrogen (S_1) is taken into the cells and supports cell growth. μ is the growth rate, π is the lipid production rate and ρ is the nitrogen uptake rate into the cells, while s_1^i and s_2^i are the concentrations of nutrients in feeds, V is the volume of the bioreactor, while D is reserved for the dilution rate which is equal to $D=(f_1^i+f_2^i)/V$ and k_m is the maintenance factor. f_1^i and f_2^i are the feed rates of glycine and glucose as the sources of nitrogen and glucose, respectively. These nutrient feed rates are available control inputs applied to the fed-batch process model. There is also an outflow (f^0) which is only used for the measurements and monitoring purposes.

Nonlinearities in the model description arise from the algae growth and nutrient consumption rate functions. The rate functions (μ, π, ρ) depend on the states of the system, which are given as follows:

$$\mu = \mu_m \frac{S_2}{K_\mu + S_2 + \frac{S_2^2}{K_{i2}}} \frac{\tilde{q}}{K_{\tilde{q}} + \tilde{q}} \exp\left(-\frac{\frac{1}{t}\int_0^t \tilde{q}\, dt}{K_{i1}}\right) \tag{7}$$

$$\pi = \pi_m \frac{S_2}{K_\pi + S_2}(1-\tilde{p}) \tag{8}$$

$$\rho = \rho_m \frac{S_1}{K_\rho + S_1} \tag{9}$$

where $\tilde{q}$ and $\tilde{p}$ are the mass fraction of nitrogen and lipid in the cells, respectively ($\tilde{q}=q/(x+p+q)$, $\tilde{p}=p(x+p+q)$). As it can be seen in Eq. (7), the growth rate depends on glucose concentrations (S_2) and nitrogen content in the cells. This dependency is described as multiplication of Michaelis–Menten kinetics for glucose and nitrogen sources. The growth rate also depends on the history of nitrogen content in the cells. Lipid production and nitrogen uptake rates are also assumed to obey Michaelis–Menten kinetics. The parameters of the microalgae model are shown in Table 1. In the experimental realization of a microalgae bioreactor, which is done by Nadadoor et al. (2012), the same states are measured using laser Raman spectroscopy.

In general, the states of the growth model can be measured offline, while for the real-time online measurements, Raman spectroscopy can be utilized. In the microalgae bioreactor used in this work, the nitrogen concentration (S_1) in the bioreactor cannot be detected directly by the Raman spectroscopy method and should be estimated on the basis of reliable real-time measurements for further regulation purposes.

3. Optimization, estimation and control methods

The main objective of this work is to demonstrate reliable lipid production maximization in the microalgae bioreactor by the application of advanced optimization, estimation and control techniques. In general, the control strategy implementation requires both full knowledge of the state trough measurement or estimation and a reliable reference trajectory. In this work, the reference trajectory is obtained by using the interior point

Table 1
Microalgae growth and lipid production model parameters.

Parameter	Value	Unit
$Y_{x/s}$	0.55	–
$Y_{p/s}$	0.34	–
$Y_{x/q}$	56.67	–
$Y_{x/p}$	11.84	–
k_m	0.19	1/d
μ_m	14.18	1/d
K_μ	8.45	g/L
$K_{\tilde{q}}$	0.0041	–
ρ_m	0.93	1/d
K_ρ	0.14	g/L
π_m	0.50	1/d
K_π	0.09	g/L
K_{i2}	49.50	g/L
K_{i1}	0.016	–

optimizer (IPOPT), and the states are estimated using a moving horizon estimator; then, with full states available, a model predictive control based strategy is used to track a desired reference trajectory. As linear MPC and MHE are used, reliable linear models are required which are achieved by the subsequent process model linearization.

3.1. Feeding strategy optimization

Microalgae lipid concentration in the bioreactor can be manipulated by determining nutrient feed rates. Glucose and glycine feed rate profiles determine the growth and lipid production trajectories. In order to obtain an optimal trajectory in the sense of lipid production, we use the IPOPT optimization method to maximize the lipid concentration in the bioreactor by manipulating nutrient input rates. The process dynamic is highly nonlinear and there are constraints on the states, process inputs and final bioreactor volume which induce model complexity that can be handled only by large-scale nonlinear optimization programs. In order to handle the nonlinearity and complexity of the model, the process model and the underlying optimization problem is formulated on the basis of the model given by Eqs. (1)–(9) as a nonlinear program. The total process time is eight days and time discretization is accurate enough, with sampling time of 30 min and the total process time of eight days that induces 768 manipulating parameters in the program. In addition, the process discretization in time will produce over 6000 equality and inequality constraints. Therefore, the optimization problem is transformed to a large-scale nonlinear program with over 6000 constraints. Due to significant complexity, conventional numerical algorithms are not able to solve a large-scale nonlinear program with enough accuracy and within a reasonable time period, and cannot to be implemented in the on-line autonomous control and monitoring. The nonlinear program is formulated as follows:

$$\xi^{k+1} = \xi^k + hf(\xi^k, u^k) \tag{10}$$

$$0 \le u^k \le u_{max} \tag{11}$$

$$0 \le \xi^k \le \xi_{max} \tag{12}$$

$$V^{t_f} = 2 \tag{13}$$

where $\xi = [S_1, S_2, x, p, q, V]$ is the state vector and ξ^k is the state vector at time k. The dynamic model of microalgae is realized with equality constraints given by Eqs. (10), (13) and inequality constraints given by Eqs. (11) and (12), which yield nonlinear program to be solved. Table 2 shows the constraints used in the optimization of lipid production. Due to the model structure, Eqs. (1)–(6), negative inputs are manifested as an increase in the lipid and biomass concentration, which means that the constraints on the inputs are necessary and will be active in the lipid production optimization by IPOPT. The other active constraint is the volume of the bioreactor which is finite. The maximum volume should not exceed 2 L. In addition, there is also a constraint on the maximum nutrient concentration to prevent dehydration of the microalgae cells. Finally, the large-scale nonlinear program is solved using AMPL as an interface and by utilization of the IPOPT solver. The resulted input feeds and state trajectories are used as a reference for the subsequent control and estimator design.

Table 2
List of constraints.

Variable	Lower bound	Upper bound	Unit
Glucose concentration (S_2)	0	50	g/L
Nitrogen feed rate (f_1^i)	0	2	mL/h
Glucose feed rate (f_2^i)	0	10	mL/h
Nitrogen feed rate change (Δf_1^i)	−0.2	0.2	mL/h
Glucose feed rate change (Δf_2^i)	−1	1	mL/h
Bioreactor volume (V_{max})	–	2	L

3.2. Model linearization

Handling a nonlinear and time-varying system by successive approximations yielding linear models requires taking account of the physical process time relevant scales and adequate and computationally reliable procedure that can be applied robustly in real-time. In particular, the microalgae process cannot be approximated by a single linear model, as the single linearized model is not able to predict the system's behaviour through the entire bioreactor single cycle run. In this work we generate a set of linearized models along the optimal trajectories obtained by the IPOPT large-scale optimization. The algorithm linearizes the mathematical model based on the states at the current time along the reference state trajectory, so that MPC and MHE are designed based on these local linear models. Local linear models are obtained by replacing the time-varying integral term in Eq. (7) by a constant parameter. This integral term depends only on the measurable states and it is numerically calculated by replacing a constant parameter, given by the following equation, at each instance of the linearization process. Therefore, we have

$$\frac{1}{t}\int_0^t \tilde{q}(t)\,dt = \frac{1}{kT}\sum_{j=1}^{k} \tilde{q}(j)T \tag{14}$$

Linearized models are accurate enough for 40 sampling times and therefore, to reduce computational effort, local linear models are used for about 2 h of the process time.

3.3. Moving horizon estimator (MHE)

Due to necessity to utilize the entire knowledge of the state in the regulation achieved by the model predictive control algorithm, non-measured bioreactor system's states (nitrogen concentration) need to be estimated accurately enough. One of suitable observers (estimators) in the presence of input and state constraints is a moving horizon estimator (MHE). As it can be inferred from the mathematical model Eqs. (1)–(6), the origin is a single stable equilibrium of the dynamical model, and the states cannot take negative values, this is imposed by a physical condition that the concentrations cannot be negative. These physical restrictions associated with the model are manifested as restrictions on the states and inputs allowed signs specified in the observer design. In addition, the limit on glucose concentration available in the feed stream, which is considered as an input constraint, encourages the usage of an observer which can handle constraints.

State estimation by the moving horizon estimator is based on an idea similar to the synthesis of model predictive control. In the moving horizon state estimation approach, the state is estimated by solving an optimization problem in an iterative manner over the moving time horizon. In comparison with the model predictive controller, where future deviations of predictions from the reference value are minimized, the MHE minimizes past deviations of the trajectory from the output measured values. The MHE estimates all states and accounts for the noise in previous N steps (estimation horizon) by optimization based on online available measurements. At each step, previous states and noises will be reconstructed such that the error between the measured outputs

and model predictions is minimized. Supposing that there is an initial guess about the states ($\bar{\xi}_{k-N-1}$) at the time $k-N-1$, the consequent states can be calculated by (Muske and Rawlings, 1993b)

$$\hat{\xi}_{k-N-1|k} = \bar{\xi}_{k-N-1|k-N} + \hat{\omega}_{k-N-1|k} \tag{15}$$

$$\hat{\xi}_{j+1|k} = A\hat{\xi}_{j|k} + Bu_j + \hat{\omega}_{j|k} \tag{16}$$

where $\hat{\xi}$ is the estimated state vector ($\xi = [S_1, S_2, x, p, q, V]^T$), $\hat{\omega}_j$ is noise estimate at the step j and $\hat{\omega}_{k-N-1|k-N}$ is reserved to modify uncertainties in the initial guess of the first state in the estimation window. The MHE problem is stated as a minimization problem of the objective function (Ψ_k) and is stated as follows:

$$\min_{\{\hat{\omega}_{k-N-1|k},\ldots,\hat{\omega}_{k-1|k}\}} \Psi_k = \hat{\omega}^T_{k-N-1|k} Q_0 \hat{\omega}_{k-N-1|k} + \sum_{j=k-N}^{k-1} \hat{\omega}^T_{j|k} Q \hat{\omega}_{j|k} + \sum_{j=k-N}^{k} \hat{e}^T_{j|k} R \hat{e}_{j|k} \tag{17}$$

s.t.:

$$\hat{\xi}_{k-N-1|k} = \bar{\xi}_{k-N-1|k} + \hat{\omega}_{k-N-1|k} \tag{18}$$

$$\hat{\xi}_{j+1|k} = A\hat{\xi}_{j|k} + Bu_j + \hat{\omega}_{j|k} \tag{19}$$

$$y_j = C\hat{\xi}_{j|k} + \hat{e}_{j|k} \tag{20}$$

where A, B, C are local linear matrices and $\hat{e}_{j/k}$ is the error between the measurements and predicted output ($\hat{e}_{j|k} = y_j - C\hat{\xi}_{j|k}$). The state estimation is subjected to the state constraints and chemically and/or biologically represented state variables cannot take negative values in Eqs. (1)–(6), which is enforced by

$$\hat{\xi}_{j|k} > 0 \tag{21}$$

Substituting $\hat{e}_{j/k}$ from Eq. (20) into Eq. (17) results in a convex quadratic program which can be solved by MATLAB or any commercial optimization package. Then, Eqs. (15) and (16) will be used to reconstruct the states.

3.4. Model predictive control (MPC)

Knowing the optimal reference trajectory and having the knowledge of all states, the MPC is designed to track the reference trajectory. Model predictive control uses the mathematical model of the process to predict future behaviour of the process and optimize the input actions in order to plan the best action to reach a desired objective. Usually, the MPC objective is to achieve regulation or trajectory tracking. The optimization objective with reference to the tracking problem is defined as (Muske and Rawlings, 1993a)

$$\min_{u_0 \ldots u_{N-1}} \Phi = \sum_{j=1}^{N} (y_j^r - C\xi_j)^T Q' (y_j^r - C\xi_j) + \sum_{j=0}^{N-1} (u_j - u_j^r)^T R' (u_j - u_j^r) \tag{22}$$

y_j^r and u_j^r are the reference trajectory and reference inputs at point j and $C\xi_j$ is the process output. Q' and R' are the positive definite penalty matrices. The optimization is also subject to the process dynamics and the constraints given by

$$\xi_{j+1} = A\xi_j + Bu_j \tag{23}$$

$$y_j = C\xi_j \tag{24}$$

$$0 \le u_j \le u_{max} \tag{25}$$

$$0 \le \xi_j \le \xi_{max} \tag{26}$$

The trajectory obtained from Eqs. (10)–(13) is used as a reference trajectory and an MPC reference tracking controller is constructed to track the reference obtained as a solution of a nonlinear program, Eqs. (10)–(13). The reference trajectory tracking control is formulated as Eqs. (22)–(26) and inputs, $[u_0 \ldots u_{N-1}]$, are determined such that the objective function, Eq. (22), is minimized. Once the sequence of optimal inputs from optimization problem is obtained, $[u_0 \ldots u_{N-1}]$, only the first input, $u^* = u_0$, is implemented and the process is iteratively repeated over a receding horizon.

The schematic of integrated estimation, control and optimization strategy is given in Fig. 1. As it can be seen, the maximization process is carried out in two steps. In an off-line optimization step, the IPOPT solver is used to determine the optimal reference trajectory that has taken into account all the present constraints on the inputs and states. In an online step, providing the reference trajectory is in place, local linear models are used by MPC to track the predetermined optimal trajectory. At the same time, MHE provides full information of the states to MPC. Both MPC and MHE account for constraints used for off-line optimization.

4. Results and discussion

Applying the above-mentioned procedure, the obtained results are shown in Table 3 along with the results from the previous

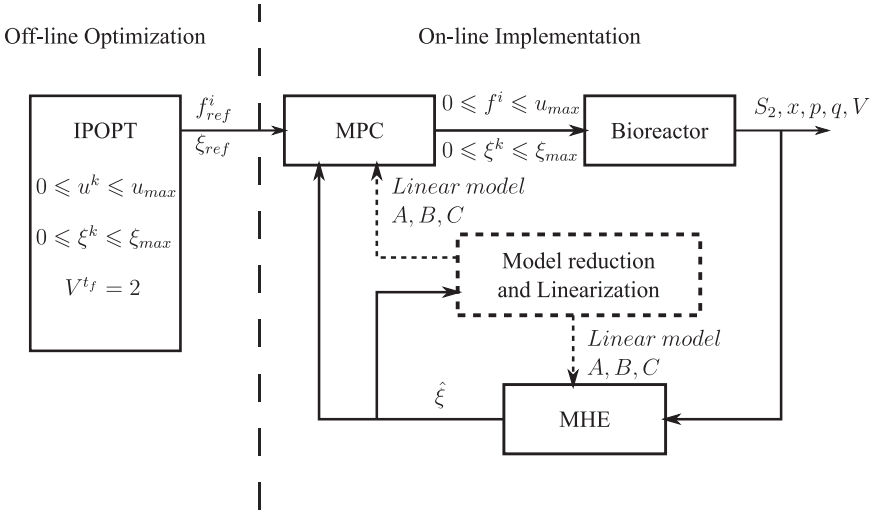

Fig. 1. Schematic of optimization, estimation and control procedure in microalgae bioreactor.

Table 3
Oil production optimization results compared to De la Hoz Siegler et al. (2012).

Parameter	De la Hoz Siegler et al. (2012)	Obtained by IPOPT optimization	Unit
Biomass concentration	144	178	g/L
Lipid Concentration	80	109	g/L
Average lipid productivity	10.3	13.8	g/L d
Maximum lipid productivity	20.1	21.3	g/L d
Average Biomass productivity	15	23	g/L d
Maximum lipid productivity	60	61.2	g/L d
Oil content	60	62	%

work of De la Hoz Siegler et al. (2012). There are significant improvements in the maximum lipid and biomass production using the suggested model predictive estimation, control and optimization strategies. Namely, the maximum magnitude of the lipid concentration achieved is 109 g/L which is 36% higher than the reported value (De la Hoz Siegler et al., 2012), while the biomass concentration increased by 24%. The improvement in biomass and lipid production is achieved taking into account all the constraints present in realistic real-time implementation. Maximum lipid and biomass productivity, which is reported in Table 3, occurs during the exponential growth and lipid accumulation period and is a characteristic of the microalgae cells growth process. These productivities are independent of the feeding strategies and can be seen as a measure of verification with respect to the previous experimental work. The achieved values for maximum productivities are within an acceptable range, with a 2% and 6% difference with respect to the reported values in De la Hoz Siegler et al. (2012). The achieved oil content of microalgae cells also shows a slight difference compared to the previous work. The same oil content of cells shows that the improvement is achieved by a higher biomass production. In other words, higher biomass production rate and the same oil content mean higher lipid production rate. As it can be also seen in Table 3, the average biomass productivity is increased by 53%, while the increase in average lipid productivity is about 34%.

Time evolution of the states and inputs is shown in Figs. 2–10, dashed and solid lines are the reference trajectories and simulation results with MPC and MHE in the absence of measurement noise, respectively, and dash-dotted lines represent the results in the presence of measurement noise. Fig. 2 shows the estimated nitrogen concentration in the bioreactor compared to its actual value. The MHE tracks the changes in nitrogen concentration (S_1) and captures its rapid increases in the bioreactor. As the time elapses, the time-varying term in Eq. (7) converges to 1 and consequently, as the time-varying term vanishes, the estimation performance becomes more efficient. The estimation error between days one and two directly affects the intracellular nitrogen concentration and as it can be seen in Fig. 8, the intracellular nitrogen concentration weakly tracks the reference trajectory in this period. The error in the estimation of nitrogen concentration directly reflects in the nitrogen uptake rate (Eq. (9)), which determines how nitrogen is up-taken into microalgae cells. The active biomass and lipid concentration in the bioreactor is shown in Figs. 3 and 4 and despite relatively poor performance of the state estimation in the first to the second day-period, the optimal control strategy tracks the reference trajectory with high efficiency. Finally, Fig. 5 shows the oil content in microalgae cells. The oil content of microalgae cells decreases during the first day due to the high growth rate, but starts to grow afterward when glucose is present and nitrogen concentration is limited.

Under an optimized feeding strategy, microalgae cells behave as expected. At low concentration of glucose and nitrogen, microalgae

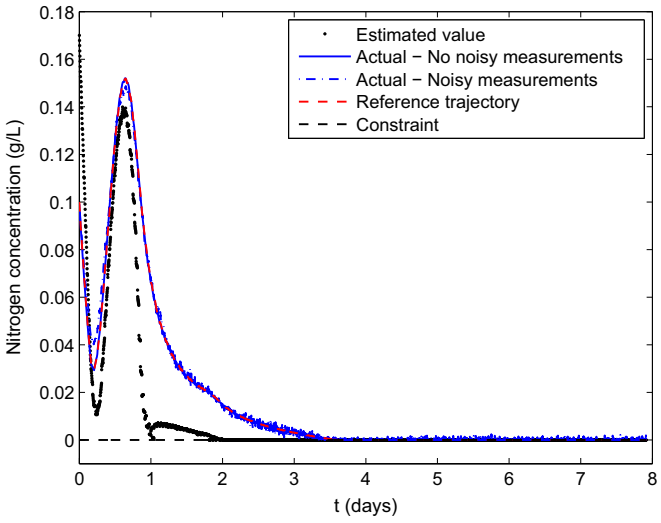

Fig. 2. Actual and estimated nitrogen concentration in the bioreactor, S_1.

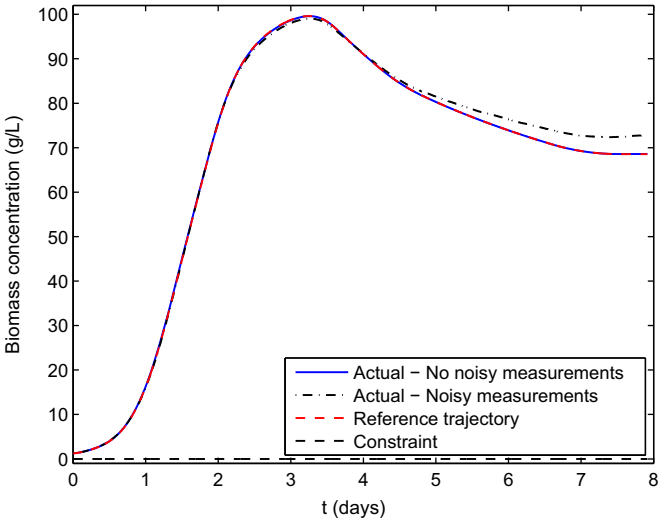

Fig. 3. Active biomass concentration in the bioreactor, x.

tend to store glucose as lipids to continue surviving in the absence of glucose. Glucose is completely consumed after the second day (Fig. 7) and it is then that the nutrition–starvation cycle starts to accumulate lipids in the cells. The optimum control strategy, as it can be seen from Figs. 2, 7, 9 and 10, is to feed microalgae to grow to as high biomass concentration as possible, when due to the craving for nitrogen, microalgae start to produce lipid.

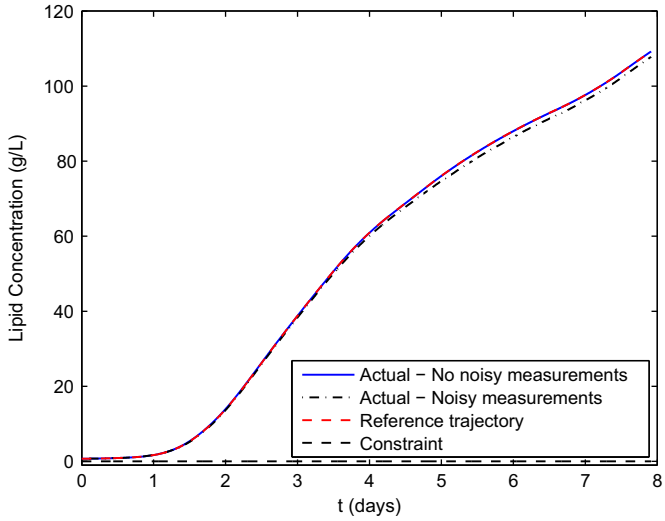

Fig. 4. Lipid concentration in the bioreactor, p, and optimal lipid concentration reference trajectory.

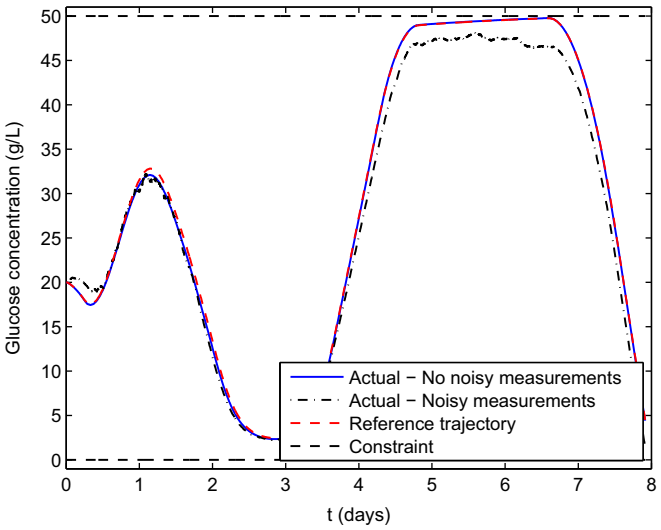

Fig. 7. Glucose concentration in the bioreactor, S_2.

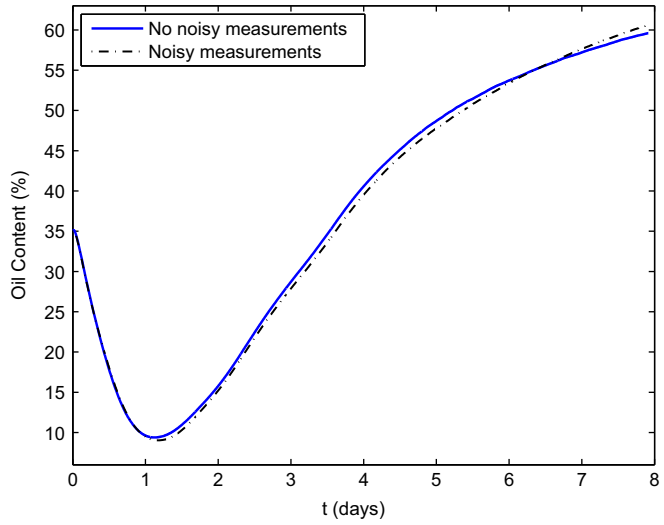

Fig. 5. Oil content in the microalgae cells.

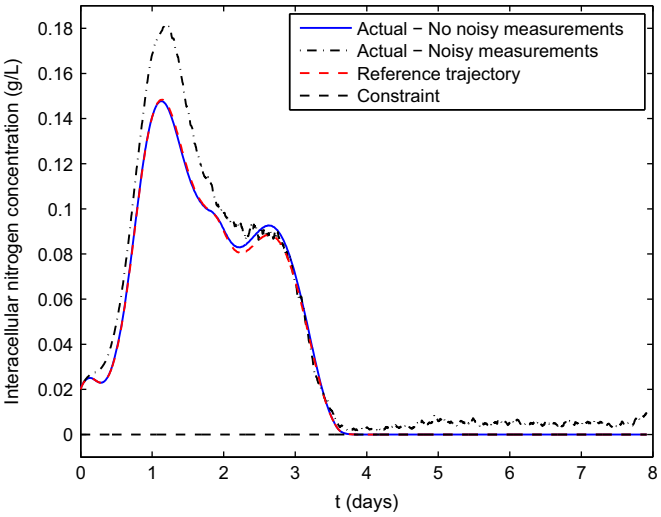

Fig. 8. Intracellular nitrogen concentration, q.

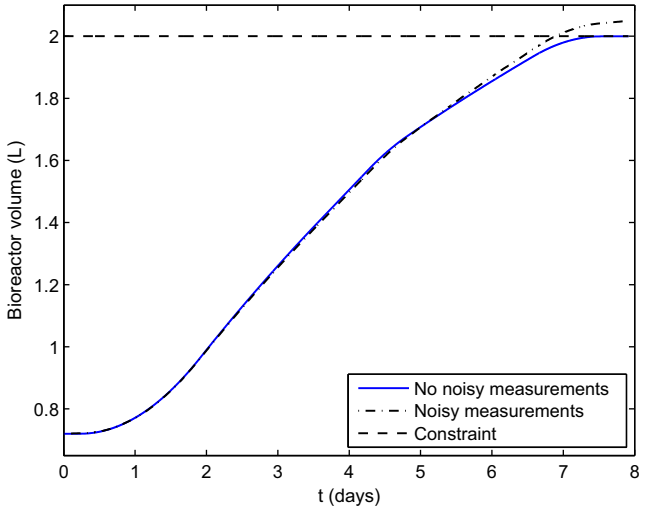

Fig. 6. Bioreactor volume, V.

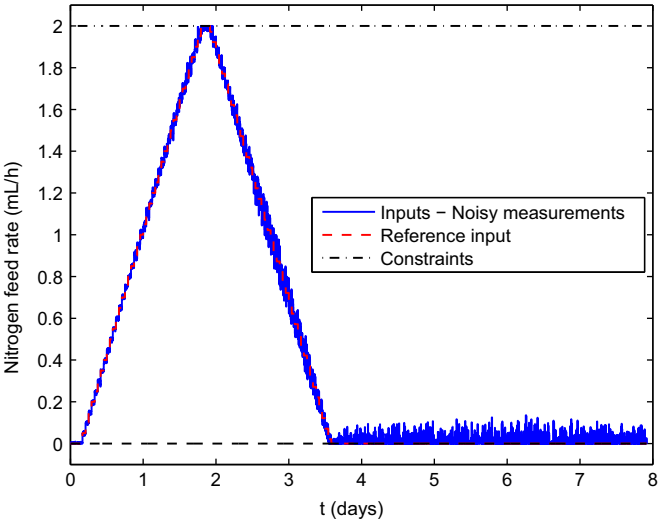

Fig. 9. Glycine feed rate (nitrogen source).

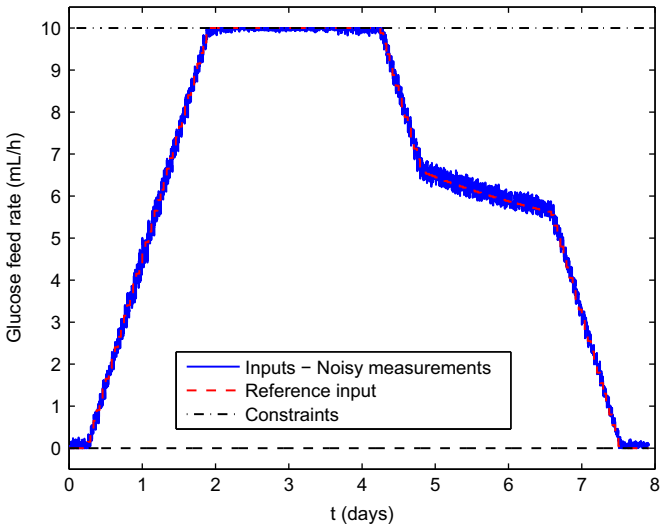

Fig. 10. Glucose feed rate (carbon source).

Nutrient feed rates (inputs) are shown in Figs. 9 and 10. Nitrogen is fed to the bioreactor only during the first three days to support algae growth and thereafter nitrogen fed is stopped to create nitrogen limited condition which is a favourable condition for lipid accumulation (Fig. 9). The nitrogen feed rate is determined by constraints, namely upper and lower limits on nitrogen feed rate and feed rate change ($\Delta f_1^i < 0.2$ mL/h and $f_1^i < 2$ mL/h). The glucose feed rate is governed by the present input and state constraints. During the first five days of operation, the glucose feed rate is governed by the constraints on the glucose feed rate change ($\Delta f_2^i < 1$ mL/h) and upper limit for the glucose feed rate ($f_2^i < 10$ mL/h). The upper limit constraint on the glucose concentration in the bioreactor (S_2) can be seen as an input constraint between days five and seven (see Figs. 7 and 10).

Under realistic operating conditions, the measurements of biological properties are highly noisy, so that the presence of noise must be considered in the estimator and controller synthesis. The simulations are repeated for the case with noisy measurements. A uniform random noise of magnitude 20% of nominal values of the states is added to the measurements. The achieved results are shown in Figs. 2–10 as dash-dotted lines. It can be seen that even with noisy measurements, the MPC tracks the reference trajectory. The feed rates in the presence of noise differ from reference inputs due to the quadratic optimization which tries to track the reference lipid production trajectory. The resulted lipid concentration in the bioreactor is slightly lower than the maximum possible lipid concentration obtained from IPOPT. Due to the noisy measurement, the reference trajectories and the inputs are not optimal and as a result, new feed rates determined by MPC cause a change in the optimal solution which then leads to a slight violation of constraint associated with the bioreactor volume (Fig. 6). However, the constraints on inputs and other states are not violated. There is an error in tracking of nitrogen (S_1) and intracellular nitrogen (q) between days one and two. The low efficiency of tracking is due to an estimation error for the nitrogen concentration (Figs. 2 and 8). The glucose concentration (S_2) has also tracking error between days four and seven (Fig. 7). The noisy measurements and the reference trajectory close to the glucose constraint give rise to an error in reference trajectory tracking of glucose concentration during the mentioned time interval (Fig. 10).

Choosing sampling time has a direct effect on the accuracy of linear models and computational effort of MPC and MHE. Shorter sampling time will make simulations more accurate, but on the other hand, for specified control and estimation horizons, smaller sampling time implies more time steps in the horizon which induces larger matrices for the evaluation of quadratic programming algorithms. Thus, there is a trade-off between accuracy and computational effort. In this work, the sampling time is chosen to be 3 min, while control and estimation horizons are 5 and 2 h, respectively.

There is a difference between trajectory tracking and simple maximizing of lipid production. The former is a reliable optimal control method which can be simply implemented to an experimental bioreactor. In the model predictive trajectory tracking, the optimization problem is convex and can be handled and implemented on the online realization of a bioreactor with inaccurate measurements. In the optimal trajectory tracking, the presence of measurement noises can be tolerated by MPC by tuning the penalty matrices. However, in the case of direct maximization of lipids, the lack of the reference trajectory will likely decrease the controller efficiency.

The faster biomass and lipid production results in low saturated fatty acid content and induces lower cold filter plugging point (CFPP) (De la Hoz Siegler et al., 2012) which is an indication of the fuel's ability to flow through a filter. Another advantage of using reference tracking is that the maintaining of the lipid production along the specified trajectory will guarantee the same lipid quality for different fed batch runs. In addition, the same quality of lipid for different fed-batch runs will reduce the cost of lipid to biofuel conversion.

5. Conclusion

Lipid production rate of the *Auxenochlorella protothecoides* microalgae bioreactor is maximized by using the IPOPT solver for constrained large-scale nonlinear program formulation. The microalgae bioreactor process dynamics model is reduced and linearized to a set of linear and time invariant models. Then by using the constrained moving horizon estimator and the set of linear models, nitrogen concentration is estimated. Finally, model predictive control is used to track the obtained reference trajectory. The optimization results show 36% increase in lipid concentration in the microalgae bioreactor. Also, The implemented MHE based state estimation and MPC reference trajectory tracking regulator not only maintain the functionality as monitoring and regulation devices in the presence of measurement noise, but show reliable performance as well.

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