Modelling and control of solar thermal system with borehole seasonal storage

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Abstract

The paper addresses the problem of controlling a solar thermal storage system with the purpose of achieving a desired thermal comfort level and energy savings. A solar thermal power plant is used for heating district houses with borehole seasonal energy storage. As the energy output from the solar thermal plant with borehole seasonal storage varies, the control system maintains the thermal comfort by using a servo controller. In this work, the modelling of the solar thermal system with borehole seasonal storage is inspired by the Drake Landing Solar Community in Okotoks, Alberta, Canada [1]. The discrete model of the integrated energy system is obtained by using energy preserving Cayley-Tustin discretization. A simple and easily realizable servo control algorithm is designed to regulate the system operating at desired thermal comfort level despite disturbances from the solar thermal plant system, the borehole geo-thermal energy storage system and/or the district heating loop system. Finally, the performance of the servo controller and frequency analysis of the plant is given in simulation results section.

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1. Introduction

The modelling and control of the solar thermal system with borehole seasonal storage is motivated by the need for accurate modelling and analysis of the state of the art community development of the Drake Landing Solar Community (DLSC) in Okotoks, Alberta, Canada [1]. The DLSC contains 52 energy-efficient houses with an innovative heating system which includes a solar thermal power plant, borehole thermal energy storage system (BTES), short term thermal storage system (STTS) and a district heating loop system. Solar thermal energy is collected through roof mounted plate collectors. A heat transfer fluid containing a high concentration of glycol is used to collect solar energy. The energy collected by the glycol loop is transferred to STTS by a heat exchanger, see Fig. 1. Depending on the season and energy requirements, the energy from the STTS can be distributed to the district heating loop or stored in the BTES. The BTES uses a grid of boreholes with single u-tube heat exchangers [2,3]. Finally, the collected energy is sent to the district heating loop system to heat the energy efficient homes.

A major step towards the completion of the control system is the development of a process model using detailed energy balances. The solar thermal plant system concentrates solar radiation using mirrors or lenses to heat a fluid [4]. The dynamic model of the solar thermal energy system is of distributed nature. However, the changes of solar radiation on warm sunny or cold cloudy days affect the dynamics of the solar thermal system. Due to this solar radiation variability, many advanced control techniques have been applied to the solar thermal energy system to account for possible problems caused by the solar radiation under variable solar energy supply [5–8].

The energy collected by a solar thermal plant system is sent to the STTS through heat exchanger. The dynamics of the heat exchanger system is distributed in nature and is modelled by the transport thermal distributed parameter system [9]. The energy from the solar thermal system is transferred to the heat exchanger system through the boundary and the counter-current flows exchange the energy, therefore, the boundary controlled system realization is considered in the modelling of the heat exchanger system.

The BTES uses a grid of boreholes with U-tube heat exchangers to preserve energy as a long-term storage device in the overall system. To fulfill energy requirements in different seasons, the BTES saves energy during the summer months by transferring available thermal energy to the ground and provides energy from the ground...
The energy balance and dynamics of the BTES is modelled as a transport thermal distributed parameter system [10,11]. In particular, when it comes to the BTES, environmental temperature fluctuations make possible sources of disturbances to the BTES system and may affect the time evolution of the model.

In the STTS, water-filled storage tanks act as a thermal buffer between the solar thermal plant system and the district heating loop system [2]. During the summer months, the hot tank utilizes thermal energy from the solar plant. When the temperature of the hot tank rises above the set-point, thermal energy from the hot tank is transferred to the BTES system. During the heating season, the hot tank charges thermal energy from both the solar plant and the BTES. Finally, the collected energy of the STTS is sent to a district heating loop system.

In order to heat the energy efficient homes in the district heating loop system, a backup gas boiler is provided to ensure that heat is available to each and every home at all times. One important performance specification is to heat homes to the prespecified temperature (which may fluctuate with seasonal changes in temperature), and therefore a controller for the natural gas boiler system is designed to track the desired temperature set-point. The temperature regulation of the solar thermal system with borehole seasonal storage is characterized by many uncertainties, such as environmental changes, occupancy status changes, and changes in the operating conditions of equipment in the building. Therefore, control systems play an important role in maintaining the performance of the systems in the presence of possible uncertainties and disturbances. The ultimate performance goal is that the proposed controller maintains the temperature at a desired set point and keeps the integrity of the energy demands in the district heating loop system.

Servo controller design is a well-know strategy that computes the required input which asymptotically attenuates error between the output and a reference trajectory or set point to zero [12,13]. One of the advantages of a servo controller is that it can account for disturbances which may affect the process. We propose a servo control system design for the solar thermal system regulation with borehole seasonal storage, which takes into account measurable disturbances, such as changes in ambient temperature and disturbance predictions, such as weather forecast that may potentially assist in the prediction of the availability of the different energy sources.

From the literature review, most of the modelling of subsystems, such as solar thermal energy system [4], heat exchanger system [9], and BTES system [10,11] are continuous and distributed in nature. In this work, in order to realize accurate modelling of the subsystems and to design a practical and usable controller, discrete models of the subsystems and a discrete controller design are developed. We utilize Cayley-Tustin time discretization which preserves the infinite-dimensional nature of the distributed parameter system [14]. This transformation preserves the energy equality among the continuous and discrete model which provides a discrete model for controller design and frequency analysis. Other model reduction technique, such as explicit Euler discretization may potentially transfer the stable continuous system into unstable discrete system or require small time steps for approximation. This proposed discretization transforms the system from a continuous to a discrete state space setting without spatial discretization and/ or any other type of spatial approximation of the distributed parameter system. In this work, according to the energy balance conservation laws, the processes in solar thermal system with borehole seasonal storage are modelled using ordinary differential equations (ODEs), hyperbolic partial differential equations (PDEs) or coupled PDEs-ODEs equations. In particular, by application of Cayley-Tustin time discretization we maintain the low dimensionality of the overall discrete model. The discrete representation of coupled partial and ordinary differential equations does not include any high order plant representation, which is contrary to the previous proposed methods [15]. In addition, a discrete infinite-dimensional representation of the system realized in this paper provides an insight into frequency response of the subsystems and that of the overall plant. This is of importance, since all well known frequency analysis methods and controller synthesis can be easily applied, and one can obtain appropriate engineering insight into...
plant operation. Finally, the controller designed for the servo problem is a discrete controller which can be easily realized and implemented in practice.

The paper is organized as follows: section 2 introduces the Cayley-Tustin time discretization. In section 3, we address the model of the solar thermal system with borehole seasonal storage and discretize the subsystems of the overall plant. Section 4 provides the servo controller design and the analysis of the system frequency response. Finally, we demonstrate the performance of the servo control formulations built in previous section through simulation studies.

2. Time discretization for linear system

According to the energy balance, the processes in the solar thermal system with borehole seasonal storage can be modelled by ordinary differential equations (ODEs), hyperbolic partial differential equations (PDEs) and/or coupled PDEs-ODEs equations. In other words, the overall system contains internally coupled linear or finite and infinite dimensional systems, see Fig. 2. The Cayley-Tustin time discretization method is applied to obtain a discrete model version which provides an insight into the subsystems’ performance and overall dynamical behaviour of the system.

2.1. Time discretization for linear infinite-dimensional system

In this section, we introduce the time discretization called the Cayley-Tustin transformation of continuous time systems to discrete time systems [14]. The linear infinite-dimensional system is described by the following state space system:

\[ \dot{x}(\zeta, t) = Ax(\zeta, t) + Bu(t), x(\zeta, 0) = x_0 \]
\[ y(t) = Cx(\zeta, t) + Du(t) \]  

where the following assumptions hold: the state \( x(\zeta, t) \in H \oplus R^n \), \( H \) is a real Hilbert space with inner product \( \cdot, \cdot \) and \( R^n \) is a real space, where \( n \) accounts for the states associated with the lumped parameter system. This state-space representation accounts for coupled infinite and finite dimensional systems. The input \( u(t) \in U \) and the output \( y(t) \in Y \), where \( U \) and \( Y \) are real Hilbert spaces; \( A:D(A);H \rightarrow H \) is the generator of a \( C_0 \)-semigroup on \( H \) and has a Yosida extension operator \( A_{-1} \); \( B \), \( C \) and \( D \) are linear operators associated with actuation and output measurement or a direct feed forward element, i.e., \( B \in L(U,H), C \subseteq L(H,Y) \) and \( D \in L(U,Y) \).

Given the time discretization parameter \( h > 0 \), the Tustin time discretization is given by Ref. [16]:

\[ \frac{x(jh) - x((j-1)h)}{h} = A \frac{x(jh) + x((j-1)h)}{2} + Bu(jh), x(0) = x_0 \]
\[ y(jh) = C \frac{x(jh) + x((j-1)h)}{2} + Du(jh) \]  

(2)

Let \( u^h/j_\sqrt{h} \) be the approximation of \( u(jh) \) and \( y^h/j_\sqrt{h} \) be the approximation of \( y(jh) \), the above set of equations yields the discrete time dynamics:

\[ \frac{x_{j+1}^h - x_j^h}{h} = A \frac{x_{j+1}^h + x_j^h}{2} + B \frac{u^h}{\sqrt{h}} x_0^h = x_0 \]
\[ \frac{y_{j+1}^h}{\sqrt{h}} = C \frac{x_{j+1}^h + x_j^h}{2} + D \frac{u^h}{\sqrt{h}} \]  

(3)

After some basic manipulation, the discrete system takes the following form:

\[ x(\zeta, k) = A_0 x(\zeta, k-1) + B_0 u(k), x(\zeta, 0) = x_0 \]
\[ y(k) = C_0 x(\zeta, k-1) + D_0 u(k) \]  

(4)

where \( \delta = 2/h, A_0, B_0, C_0 \) and \( D_0 \) are discrete time linear system operators, given by:

\[ \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} \delta - A^{-1} A & \sqrt{2}\delta \delta - A^{-1} B \\ \sqrt{2}\delta \delta - A^{-1} C & G(\delta) \end{bmatrix} \]  

(5)

where \( G(\delta) \) denotes the transfer function of the system from input to the output and it is defined as \( G(\delta) = C(\delta - A)^{-1} B + D \).

In the most general case, Eq. (1) can be extended by introducing the affine disturbance input, which leads to the following form:

\[ \dot{x}(\zeta, t) = Ax(\zeta, t) + Bu(t) + Ed(t), x(\zeta, 0) = x_0 \]
\[ y(t) = Cx(\zeta, t) + Du(t) + Fd(t) \]  

(6)

where \( E \in L(R^n,H) \) and \( F \in L(R^n,Y) \) are linear operators. The corresponding discrete operators are \( E_d = \sqrt{2}\delta(\delta - A^{-1})^{-1} E \) and \( F_d = C(\delta - A)^{-1} E + F \).

Remark 1. Discrete operator \( A_d \) can be expressed as \( A_d = [\delta - A]^{-1} [\delta + A] = -I + 2\delta(\delta - A)^{-1} \), here \( I \) is the identity operator.

Proof: In order to demonstrate the results in Remark 1, one can show that:

\[ A_d(\cdot) = [\delta - A]^{-1} [\delta + A](\cdot) \]
\[ = \frac{\delta + A}{\delta - A}(\cdot) \]
\[ = \left[ -I + 2\delta \delta - A^{-1} \right](\cdot) \]

Remark 2. The Cayley-Tustin transform maps infinite-dimensional system from continuous time to discrete time without spatial approximation. The novelty of using Cayley-Tustin time discretization is that this implicit method can be applied freely with larger time steps for time integration compared to the explicit methods, such as explicit Euler and/or Runge-Kutta method.

In the next section, we apply the Cayley-Tustin discretization described above to the solar thermal system with borehole seasonal storage.
3. Model formulation and time discretization

3.1. Overview of solar thermal system with borehole seasonal storage

The solar thermal system with borehole seasonal storage modelled in this work uses the solar thermal system, heat exchanger, BTES system, STTS system, natural gas system and the district heating loop system, see Fig. 2. The thermal energy transfers from the solar thermal system to the STTS system through a heat exchanger. The BTES system stores thermal energy to the STTS system directly. Then, the STTS system provides thermal energy to the district heating loop system. Finally, the inlet to the district heating loop system is maintained at a reference temperature through the control of the natural gas system.

In this work, the solar thermal system and the BTES system are described by coupled PDEs-ODEs equations. The contraflow heat exchanger is modelled by a series of first order hyperbolic PDEs with consideration of boundary inputs. The STTS and the natural gas system are represented by ODE equations. In this section, we introduce the modelling of these subsystems and discretize the subsystems with the Cayley-Tustin transformation preserves system energy.

**Remark 3.** The system modelling in this work does not consider irreversible processes in thermal dynamic system representation. The time discretization of the closed-loop system has no modelling error associated with spatial domain discretization since Cayley-Tustin transformation preserves system energy.

3.2. Solar thermal energy system

The solar thermal energy system uses a plate collector to focus solar radiation onto the absorber pipe [4]. The energy balance of the flow in the solar collector is given as follows:

\[ \frac{\partial \phi}{\partial t} = -C_{\text{p}g}F_{\text{solar}} \cdot \frac{\partial T_{\text{solar}}}{\partial z} + h_p P_{\text{A}}(T_A - T_{\text{solar}}) \]  

(7)

and the energy balance of the absorber is:

\[ \rho_A C_{\text{p}A} \frac{dT_A}{dt} = h_p P_{\text{A}}(T_{\text{solar},in} - T_A) + Q_{\text{sol}} w \]

(8)

The description of the system variables is shown in Table 1.

One can apply appropriate non-dimensional transformation of Eqs. (7) and (8), so that the following states \( x_1 = \frac{T_A - T_{\text{ref}}}{T_{\text{ref}}, t_2 = \frac{T_{\text{solar}} - T_{\text{ref}}}{T_{\text{ref}}} \), and input \( u_1 = \frac{Q_{\text{sol}}}{w} \) are obtained. Here \( T_{\text{ref}} \) is the reference temperature and \( Q_{\text{ref}} \) is the reference heat flux. The parameters of the system are \( \alpha_1 = \frac{F_{\text{solar}}}{\rho g w}, \beta_1 = \frac{h_p S_{\text{sol}}}{\rho A C_{\text{p}A} T_{\text{ref}}} \), \( \beta_2 = \frac{h_p S_{\text{sol}}}{\rho A C_{\text{p}A} T_{\text{ref}}} \), and \( \gamma_1 = \frac{Q_{\text{sol}} w}{\rho A C_{\text{p}A} T_{\text{ref}}} \).

**Table 1** Parameters of the solar system used to model Eqs. (7) and (8).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{solar}} )</td>
<td>K</td>
<td>Flow temperature outside of solar system</td>
</tr>
<tr>
<td>( T_A )</td>
<td>K</td>
<td>Temperature of the absorber</td>
</tr>
<tr>
<td>( F_{\text{solar}} )</td>
<td>kg/s</td>
<td>Flow rate of the solar system</td>
</tr>
<tr>
<td>( C_{\text{p}g} )</td>
<td>J/kg K</td>
<td>Heat capacity of the glycol in the solar system</td>
</tr>
<tr>
<td>( C_{\text{p}A} )</td>
<td>J/kg K</td>
<td>Pipe heat capacity</td>
</tr>
<tr>
<td>( Q_{\text{sol}} )</td>
<td>W/m²</td>
<td>Solar system heat flux</td>
</tr>
<tr>
<td>( m )</td>
<td>m</td>
<td>Solar collector width</td>
</tr>
<tr>
<td>( \rho g )</td>
<td>kg/m³</td>
<td>Density of glycol flow</td>
</tr>
<tr>
<td>( \rho_A )</td>
<td>kg/m³</td>
<td>Density of pipe</td>
</tr>
<tr>
<td>( S_{\text{sol}} )</td>
<td>m²</td>
<td>Area of glycol flow</td>
</tr>
<tr>
<td>( h_p )</td>
<td>m²/W</td>
<td>Convective heat transfer coefficient for the pipe</td>
</tr>
<tr>
<td>( \mu A )</td>
<td>m</td>
<td>Absorber pipe perimeter</td>
</tr>
</tbody>
</table>

Therefore, the solar collector system can be described by the following coupled PDE and ODE system:

\[ \frac{\partial x_1}{\partial t} = -\alpha_1 \frac{\partial x_1}{\partial z} + \beta_1 (x_2 - x_1) \]

\[ \frac{dx_2}{dt} = \beta_2 (x_{1in} - x_2) + \gamma_1 u_1 \]

(9)

\[ y_1(t) = x_1(L, t) \]

By considering steady state conditions, one can obtain the following linear system by applying \( x_1(\zeta, t) = x_{1ss}(\zeta) + x_1(\zeta, t), x_2(\zeta, t) = x_{2ss}(\zeta) + x_2(\zeta, t), \) and \( u_1(t) = u_{1ss} + u_1(t) \):

\[ \frac{\partial x_1}{\partial t} = -\alpha_1 \frac{\partial x_1}{\partial z} + \beta_1 (\bar{x}_2 - \bar{x}_1), x_{1in} = \bar{x}_1(0, t) \]

\[ \frac{dx_2}{dt} = \beta_2 (x_{1in} - \bar{x}_2) + \gamma_1 \bar{u}_1 \]

(10)

\[ \bar{y}_1(t) = \bar{x}_1(L, t) \]

and we assume \( x_1(0, t) \) operates around steady state, thus \( \bar{x}_{1in} = \bar{x}_1(0, t) = 0 \).

The discrete system can be obtained by using the time discretization method described in Eqs. (4) and (5). According to Eq. (4), the resolvent of the system is calculated by using Laplace transform and the following representation: \( X_1(\zeta, t) = \left[ \bar{x}_1(\zeta, t), \bar{x}_2(\zeta, t) \right] \).

\[ U_1(t) = \bar{u}_1(t), \quad A_1 = \begin{bmatrix} -\alpha_1 \frac{\partial}{\partial z} - \beta_1 & \beta_1 \\ 0 & -\beta_2 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 0 \\ \gamma_1 \end{bmatrix} \]

and \( C_1 = \begin{bmatrix} C_\zeta & 0 \end{bmatrix} \), then the operator \( C(\zeta) = \int_0^\zeta \delta(\zeta - \lambda) \, d\lambda \) is \( f(L) \). Thus, the solar collector system can be expressed as:

\[ X_1(t) = A_1 X_1(t) + B_1 U_1(t) \]

\[ Y_1(t) = C_1 X_1(t) \]

(11)

From Eq. (11), one can obtain the Laplace transformation with the mild assumption that \( \alpha_1 = 1 \) (in general even \( \alpha_1(\zeta) \) can be considered):

\[ s\bar{x}_1(\zeta, s) - \bar{x}_1(0, 0) = -\alpha_1 \frac{\partial \bar{x}_1(\zeta, s)}{\partial \zeta} + \beta_1 [\bar{x}_2(s) - \bar{x}_1(\zeta, s)] \]

(12)

\[ s\bar{x}_2(s) - \bar{x}_2(0) = -\beta_2 \bar{x}_2(s) \]

Solving the above set of equations, the resolvent of the operator \( A_1 \) is expressed as:

\[ R(s, A_1) = [sI - A_1^{-1}] X_1(0, 0) = [R_{11} \ R_{12} \ R_{21} \ R_{22}] X_1(\zeta, 0) \]

(13)

where \( R_{11} = \int_0^\zeta \delta(\zeta - \lambda) \, d\lambda \) and \( R_{12} = \int_0^\zeta \psi(\zeta, s) \, d\lambda \), \( R_{21} = 0 \) and \( R_{22} = \frac{s}{s + \beta_2} \).

Finally, the discrete system can be expressed as:

\[ X_1(k) = A_{d1} X_1(k - 1) + B_{d1} U_1(k) \]

\[ Y_1(k) = C_{d1} X_1(k - 1) + C_{d1} U_1(k) \]

(14)

here, the discrete operators \( A_{d1}, B_{d1}, C_{d1} \) and \( D_{d1} \) are given directly as follows:

\[ A_{d1}(-) = \left[ -I + 2\delta \int_0^\zeta \delta(\zeta - \lambda) \, d\lambda \right] (-) \]

\[ A_{d1}(+) = \begin{bmatrix} A_{d1-11} & A_{d1-12} \\ A_{d1-21} & A_{d1-22} \end{bmatrix} (+) \]

where \( A_{d1-11} = -(-) + 2\delta \int_0^\zeta \delta(\zeta - \lambda) \, d\lambda \).
3.3. Borehole thermal energy storage system

The borehole thermal energy storage system uses a grid of boreholes with U-tube heat exchangers [11]. The energy balance of the flow in the U-tube heat exchanger is given as:

\[
\rho H_o C_{ph,o} \frac{dT_{borehole}}{dt} = -C_{ph,o} \frac{dT_{borehole}}{dz} + h_W P_W (T_{borehole-in} - T_W) + Q_{borehole}(t) \tag{15}
\]

The energy balance of the pipe wall is:

\[
\frac{dT_{borehole}}{dz} = \frac{Q_{borehole}}{\rho h_{ph,wall}} - \frac{dT_{borehole}}{dz} + \frac{kl}{C_{ph,wall}} [T_{borehole-in} - T_{borehole}] + \frac{Q_{borehole}}{\rho h_{ph,wall}} \tag{16}
\]

the description of the system variables is shown in Table 2.

We consider the following change of variables with states

\[
X_3 = \frac{T_{borehole}}{T_e} - 1, \quad X_4 = \frac{Q_{borehole}}{Q_{max}}, \quad \text{and} \quad u_2 = \frac{Q_{borehole}}{Q_{max}}. \tag{17}
\]

The system parameters are

\[
\alpha_3 = \frac{P_{max}}{P_{borehole}} \quad \beta_3 = \frac{h_W P_W}{h_{ph,wall} P_{borehole}} \quad \beta_4 = \frac{h_W P_W}{h_{ph,wall} P_{borehole}} \quad \text{and} \quad \gamma_2 = \frac{Q_{borehole}}{Q_{max}}. \tag{18}
\]

By applying linearization around the steady state of interest, the BTES system is described by the following coupled PDE and ODE:

\[
\frac{d\tilde{x}_3}{dt} = -\alpha_3 \frac{d\tilde{x}_3}{dx} + \beta_3 (\tilde{x}_4 - \tilde{x}_3), \quad \tilde{x}_3(0,t) = 0. \tag{19}
\]

and we assume that \(x_3(0,t)\) operates around the steady state of interest, thus \(\tilde{x}_3(0,t) = 0\). By considering

\[
X_2(\zeta,t) = \begin{bmatrix} \tilde{x}_3(\zeta,t) \tilde{x}_4(t) U_2(t) \end{bmatrix}, \quad A_2 = \begin{bmatrix} -\alpha_3 & -\beta_3 & \beta_3 \\ 0 & -\beta_4 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \gamma_2 \end{bmatrix}
\]

and \(C_2 = [C, 0]\), here operator \(Cf(\zeta) = \int_0^\zeta f(\zeta) \delta(\zeta - L)\). Finally, the BTES system can be expressed as:

\[
X_2(t) = A_2 X_2(t) + B_2 U_2(t) \quad Y_2(t) = C_2 X_2(t) \tag{20}
\]

and discrete system can be expressed as:

\[
X_2(k) = A_2 X_2(k - 1) + B_2 U_2(k) \quad Y_2(k) = C_2 X_2(k - 1) + D_2 U_2(k) \tag{21}
\]

The model of BTES system is similar to the model of the solar collector system, thus, the expression of a discrete BTES system is similar to the solar thermal energy system with different parameters.

3.4. System of heat exchanger

The heat exchanger in the solar thermal system is a counter-current heat exchanger which is modelled by a set of coupled first-order hyperbolic partial differential equations [9]. Despite the non-linearity of the controlled system, an explicit characterization of the equilibrium profiles can be given. As a consequence, the linearized system around an equilibrium profile is obtained as a linear infinite dimensional time-invariant system.

According to the heat exchange balance, we obtain the following differential equations for the heat exchanger HX-1:

\[
\frac{dT_{HX-11}(\zeta,t)}{dt} = \frac{Q_{HX-11}(\zeta,t)}{\rho h_{HX-11}} - \frac{dT_{HX-11}(\zeta,t)}{dz} - \frac{kl}{C_{ph,o} P_{HX-11}} [T_{HX-11}(\zeta,t) - T_{HX-12}(\zeta,t)] \tag{22}
\]

\[
\frac{dT_{HX-12}(\zeta,t)}{dt} = \frac{Q_{HX-12}(\zeta,t)}{\rho h_{HX-12}} + \frac{dT_{HX-12}(\zeta,t)}{dz} + \frac{kl}{C_{ph,o} P_{HX-12}} [T_{HX-11}(\zeta,t) - T_{HX-12}(\zeta,t)] \tag{23}
\]
Let \( \sim \) boundary actuation to in-domain is to apply state transformation. and transfer the boundary applied disturbance to the in-domain PDEs:

\[
\begin{align*}
\theta \frac{\partial \psi}{\partial t} & = \alpha_5 F_1 \frac{\partial \psi}{\partial \zeta} - \beta_5 \psi_0 \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \psi_0 \frac{\partial \psi}{\partial \zeta} - \alpha_6 \theta \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \psi_0 \frac{\partial \psi}{\partial \zeta} \\
& = \theta + \theta_0 \psi \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \psi_0 \frac{\partial \psi}{\partial \zeta} \\
& = \theta + \theta_0 \psi \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \psi_0 \frac{\partial \psi}{\partial \zeta}
\end{align*}
\]

Table 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of hot flow in the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow in the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of hot flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
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<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow into the HX-1 system</td>
</tr>
<tr>
<td>( T_{\text{HX-1}} )</td>
<td>K</td>
<td>Temperature of cold flow out of the HX-1 system</td>
</tr>
</tbody>
</table>

The linearized heat exchanger system is described by the following hyperbolic PDEs:

\[
\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial t} - \beta_5 \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \frac{\partial \psi}{\partial \zeta} - \alpha_6 \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \frac{\partial \psi}{\partial \zeta} = \theta + \theta_0 \psi \frac{\partial \psi}{\partial \zeta} + \frac{\partial \psi}{\partial \zeta} \psi_0 \frac{\partial \psi}{\partial \zeta}
\]

It is important to note that the linearized system around an equilibrium point is governed by the above equations with \( \psi_3(t) \) replaced by Ref. \( \psi_3(s) \).

The heat exchanger transfers the energy from the solar system to the hot tank system, the flow into the state \( \psi_3 \) is the flow out of the solar system which enters the heat exchanger at \( \zeta = L \), thus, \( \psi_3(L, t) = \psi_1(L, t) \), see Fig. 2. The flow into the heat exchanger state \( \psi_3 \) at \( \zeta = 0 \) is the flow out of the cold tank system. Since the cold tank is at the reference temperature, one can obtain \( \psi_3(0, t) = 0 \).

Since the heat exchanger system is potentially exposed to boundary disturbances, the boundary conditions need to be adequately considered in this coupled hyperbolic PDEs system. In the ensuing section, we accurately account for boundary influence and transfer the boundary applied disturbance to the in-domain disturbance.

The standard methodology to accurately account for transfer of boundary actuation to in-domain is to apply state transformation. Let \( \psi_3(L, t) = \psi_3(L, t) + B_3(\zeta) \), then \( \psi_3(L, t) = 0, B_3(L) = 1 \), \( \psi_3(0, t) = \psi_3(0, t) + B_3(0) = 1 \). With the assumptions \( \alpha_3 F_1 = 1, \alpha_6 \psi_3(s) = 1, \gamma_3 = \alpha_6 \psi_3(s) \), the above system becomes:

\[
\begin{align*}
\frac{\partial \psi_3}{\partial t} & = \frac{\partial \psi_3}{\partial t} - \beta_5 \frac{\partial \psi_3}{\partial \zeta} + \frac{\partial \psi_3}{\partial \zeta} \frac{\partial \psi_3}{\partial \zeta} - \alpha_6 \frac{\partial \psi_3}{\partial \zeta} + \frac{\partial \psi_3}{\partial \zeta} \frac{\partial \psi_3}{\partial \zeta} = \theta + \theta_0 \psi_3 \frac{\partial \psi_3}{\partial \zeta} + \frac{\partial \psi_3}{\partial \zeta} \psi_0 \frac{\partial \psi_3}{\partial \zeta} + B_3(\zeta) \frac{\partial \psi_3}{\partial \zeta} + B_3(\zeta) \frac{\partial \psi_3}{\partial \zeta} \frac{\partial \psi_3}{\partial \zeta}
\end{align*}
\]

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can be expressed as:

\[ Z_3(t) = A_{23}Z_3(t) + B_{23}U_3(t) + E_{23}G_3(t) \]

Applying Laplace transform with the boundary conditions 
\[ z_0(t_0) = 0 \text{ and } z_0(t_0) = 0 \],
the resolvent of the operator \( A_{23} \) can be expressed as follows:

\[ R(s, A_{23}) = \left| sI - A_{23} \right|^{-1} Z_3(0, 0) = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ 0 & 0 & R_{33} & 0 \\ 0 & 0 & 0 & R_{44} \end{bmatrix} \]

where \( R_{ij}(s) \) are defined in Appendix.

The discrete system is expressed as:

\[ Z_3(k) = A_{d23}Z_3(k - 1) + B_{d23}U_3(k) + E_{d23}G_3(k) \]

The discrete operators in the above equation are:

\[ A_{d23} = \sqrt{2}A_{23} \quad \text{and} \quad B_{d23} = A_{23} \quad \text{and} \quad E_{d23} = A_{23} \]

The original states are obtained by the following transform:

\[ \begin{bmatrix} x_3(t, k) \\ x_6(t, k) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left[ B_3 + B_6 \right] x_3(L, k) \\ \frac{1}{2} \left[ B_3 + B_6 \right] x_6(0, k) \end{bmatrix} \]

3.5. Short term thermal storage system

3.5.1. Hot tank system

The mass balance and energy balance of the hot tank system is modelled by the following equations [15]:

\[ \frac{d\text{ht}}{dt} = F_{\text{ht}-1} + F_{\text{ht}-2} - F_{\text{ht}-3} \rho_{ht} C_{ht} q_{ht} \frac{d\text{ht}}{dt} = \rho_{ht} C_{ht} q_{ht} (F_{\text{ht}-1} + F_{\text{ht}-2} - F_{\text{ht}-3}) \]

where \( F_{\text{ht}-1} \) and \( F_{\text{ht}-2} \) are flow rates from the heat exchanger system and the BTES system, \( F_{\text{ht}-3} \) is flow rate out of the hot tank and \( F_{\text{ht}-3} = \frac{Q}{m_{ht}} \).

The following change of variables is considered, states \( x_7 = \frac{b_{ht}}{K} \), and \( x_9 = \frac{m_{ht}}{K} \), and inputs \( u_4 = \frac{F_{ht}}{K} \), \( u_5 = \frac{q_{ht}}{K} \) and \( u_6 = \frac{q_{ht}}{K} \). Disturbances \( x_{81in} = \frac{F_{ht}}{T_{ht}-1} \), and \( x_{82in} = \frac{F_{ht}}{T_{ht}-1} \), and parameter \( \beta_7 = \frac{F_{ht}}{K_{ht}} \), the hot tank system is described by the following ODEs:

\[ \frac{dx_7}{dt} = \beta_7 (u_4 + u_5 - u_6) \]

The following table (Table 4) includes parameters of the hot tank system used to model Eq. (27).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{ht} )</td>
<td>m</td>
<td>Height of flow in the hot tank</td>
</tr>
<tr>
<td>( T_{\text{ht}-1} )</td>
<td>K</td>
<td>Temperature of flow from the HTX-1 system</td>
</tr>
<tr>
<td>( T_{\text{ht}-2} )</td>
<td>K</td>
<td>Temperature of flow from the borehole system</td>
</tr>
<tr>
<td>( T_{\text{ht}-3} )</td>
<td>K</td>
<td>Temperature of flow to the natural gas boiler system</td>
</tr>
<tr>
<td>( F_{\text{ht}-1} )</td>
<td>kg/s</td>
<td>Flow rate of flow from the HTX-1 system</td>
</tr>
<tr>
<td>( F_{\text{ht}-2} )</td>
<td>kg/s</td>
<td>Flow rate of flow from the borehole system</td>
</tr>
<tr>
<td>( F_{\text{ht}-3} )</td>
<td>kg/s</td>
<td>Flow rate of flow to the natural gas boiler system</td>
</tr>
<tr>
<td>( h_{ht} )</td>
<td>m²</td>
<td>Hot tank area</td>
</tr>
</tbody>
</table>

3.5.2. Natural gas boiler system

The mass balance and energy balance of the natural gas boiler system is modelled by the following equations [15]:

\[ \frac{dQ}{dt} = \rho_{boiler} C_{boiler} q_{boiler} \frac{dQ}{dt} = \rho_{boiler} C_{boiler} q_{boiler} (a_{boiler} + b_{boiler} q_{boiler} + c_{boiler} \frac{dQ}{dt}) \]

The following change of variables is considered, states \( x_7 = \frac{b_{boiler}}{K} \), and \( x_9 = \frac{m_{boiler}}{K} \), and inputs \( u_1 = \frac{q_{boiler}}{K} \), \( u_2 = \frac{q_{boiler}}{K} \) and \( u_3 = \frac{q_{boiler}}{K} \). Disturbances \( x_{81in} = \frac{F_{boiler}}{T_{boiler}-1} \), and \( x_{82in} = \frac{F_{boiler}}{T_{boiler}-1} \), and parameter \( \beta_7 = \frac{F_{boiler}}{K_{boiler}} \), the natural gas boiler system is described by the following ODEs:

\[ \frac{dx_7}{dt} = \beta_7 (u_1 + u_2 - u_3) \]

The following table (Table 5) includes parameters of the natural gas boiler system used to model Eq. (32).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{boiler}} )</td>
<td>K</td>
<td>Temperature of flow out of the boiler system</td>
</tr>
<tr>
<td>( F_{\text{boiler}} )</td>
<td>kg/s</td>
<td>Boiler system flow rate</td>
</tr>
<tr>
<td>( q_{\text{boiler}} )</td>
<td>W/m²</td>
<td>Boiler system collected energy</td>
</tr>
<tr>
<td>( h_{\text{boiler}} )</td>
<td>m²</td>
<td>Boiler system area</td>
</tr>
</tbody>
</table>
Table 6  Parameters of district heating loop system used to model Eq. (35).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_district</td>
<td>K</td>
<td>Temperature of flow out of the district system</td>
</tr>
<tr>
<td>F_district</td>
<td>kg/s</td>
<td>Flow rate of the district system</td>
</tr>
<tr>
<td>Q_district</td>
<td>W/m²</td>
<td>Heat flux of the district system</td>
</tr>
<tr>
<td>S_district</td>
<td>m²</td>
<td>Area of water flow</td>
</tr>
<tr>
<td>w</td>
<td>m</td>
<td>District system width</td>
</tr>
</tbody>
</table>

\[ X_4(k) = A_{d4} X_4(k - 1) + B_{d4} U_4(k) + E_{d4} G_4(k) \]
\[ Y_4(k) = C_{d4} X_4(k - 1) + D_{d4} U_4(k) + F_{d4} G_4(k) \]  

\[
A_{d4} = \begin{bmatrix} \frac{2\delta}{\delta + \beta_7 K_1} & 0 \\ 0 & \frac{2\delta}{\delta + \beta_7 K_1} \end{bmatrix},
\]

\[
B_{d4} = \sqrt{2\delta} \begin{bmatrix} \frac{\beta_7}{\delta + \beta_7 K_1} & \frac{\beta_7}{\delta + \beta_7 K_1} \\ \frac{\beta_7}{\delta + \beta_7 K_1} & \frac{\beta_7}{\delta + \beta_7 K_1} \end{bmatrix},
\]

\[
C_{d4} = \sqrt{2\delta} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},
\]

\[
D_{d4} = \begin{bmatrix} \frac{\beta_7}{\delta + \beta_7 K_1} & \frac{\beta_7}{\delta + \beta_7 K_1} \\ \frac{\beta_7}{\delta + \beta_7 K_1} & \frac{\beta_7}{\delta + \beta_7 K_1} \end{bmatrix},
\]

\[
E_{d4} = \sqrt{2\delta} \begin{bmatrix} \frac{\beta_7 u_4}{x_7_{ds}} & \frac{\beta_7 u_4}{x_7_{ds}} \\ \frac{\beta_7 u_4}{x_7_{ds}} & \frac{\beta_7 u_4}{x_7_{ds}} \end{bmatrix},
\]

\[
F_{d4} = \begin{bmatrix} \frac{\beta_7 u_4}{x_7_{ds}} & \frac{\beta_7 u_4}{x_7_{ds}} \\ \frac{\beta_7 u_4}{x_7_{ds}} & \frac{\beta_7 u_4}{x_7_{ds}} \end{bmatrix},
\]

\[ 3.5.2. \text{Cold tank system} \]

In Fig. 2, the flow coming into the cold tank in F_{CT} is from the district heating loop system. The flows out of the cold tank are linked to the solar thermal system and the BTES system, which are F_{CT2} and F_{CT3}. Here, we assume that the flow temperature of the cold tank is at reference environment temperature, which is T_{CT2} - T_{CT3} = T_{CT} - T_r. Thus, the disturbances to the solar thermal system and the BTES system are considered as zero. In simulations studies, we do not model the cold tank system.

\[ 3.6. \text{Natural gas boiler system} \]

The energy balance of the flow in the natural gas boiler system is given as follows:
\[
\rho_{H_2O} C_{H_2O} V_{boiler} \frac{dT_{boiler}}{dt} = C_{H_2O} F_{boiler} (T_{boilerin} - T_{boiler}) + Q_{boiler}\]

\[
= \frac{T_{boilerin} - T_r}{T_r - T_{boilerin}}
\]

The description of the system variables is shown in Table 5.

Let us consider the following change of variables: state \( x_9(t) = \frac{x_9(t) - T_r}{T_r - T_{boilerin}} \), input \( u_7(t) = \frac{Q_{boiler}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \), and parameters \( \beta_9 = \frac{F_{boiler}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \) and \( \gamma_9 = \frac{Q_{boiler} - H_{9}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \).

The linearized natural gas system is described by the following ODE:
\[
\frac{dx_9(t)}{dt} = \beta_9 (x_9(t) - x_9(t)) + \gamma_9 u_7(t)
\]
\[
\dot{y}_9(t) = x_9(t)
\]

here, the flow into the natural gas boiler system is the flow out of the hot tank system, so \( x_{ghin} = x_9 \).

The discrete gas boiler system can be expressed as:
\[
\dot{x}_9(k) = A_{d9} x_9(k - 1) + B_{d9} u_7(k) + E_{d9} G_9(k)
\]
\[
y_9(k) = C_{d9} x_9(k - 1) + D_{d9} u_7(k) + F_{d9} G_9(k)
\]

\[ 3.7. \text{District heating loop system} \]

The district heating loop system is modelled as a hyperbolic PDE system with the heat sink \( Q_{district} \):

\[
\rho_{H_2O} C_{H_2O} T_{district} \frac{\partial T_{district}}{\partial t} = -C_{H_2O} F_{district} \frac{\partial T_{district}}{\partial \xi} - Q_{district} W
\]

The description of the system variables is shown in Table 6.

The dimensionless system is obtained by considering the following change of variables: state \( x_{10} = \frac{T_{district} - T_r}{T_r} \), input \( u_7 = \frac{Q_{boiler}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \), and parameters \( \alpha_{10} = \frac{F_{boiler}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \) and \( \gamma_{10} = \frac{Q_{boiler} - H_{10}}{\rho_{H_2O} C_{H_2O} V_{boiler}} \).

By applying linearization, the district heating loop system becomes:

\[
\frac{\partial x_{10}(t)}{\partial t} = -\alpha_{10} \frac{\partial x_{10}(t)}{\partial \xi} - \gamma_{10} u_7
\]
\[
\dot{y}_{10}(t) = x_{10}(L_t)
\]

The flow into the district heating loop system is the flow out of the natural gas boiler system, thus, \( x_{10}(0,t) = x_9(t) \). With the consideration of boundary disturbance, let \( x_{10}(\xi,t) = x_{10}(\xi,t) + b_{10}(\xi) \dot{x}_{10}(0,0) + \dot{b}_{10}(\xi) \dot{x}_{10}(0,0) \), then \( x_{10}(0,0) = 0, b_{10}(t) = 1 \). With the assumption \( \alpha_{10} = 1 \), the above system becomes:

\[
\frac{\partial x_{10}(\xi,t)}{\partial t} = -\alpha_{10} \frac{\partial x_{10}(\xi,t)}{\partial \xi} - \gamma_{10} u_7
\]
\[
\dot{y}_{10}(t) = x_{10}(L_t) + b_{10}(L_t) \dot{x}_{10}(0,0)
\]

With the assumption \( \alpha_{10} = 1 \), one can obtain the constant function \( b_{10}(\xi) = 1 \). The extension of the system can be expressed as follows:

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In this section, the discrete state space settings of the solar thermal system can be directly obtained. The original state can be obtained by the transformation 

\[
\begin{bmatrix}
\dot{z}_{10}(\zeta, t) \\
\dot{x}_{10}(0, t)
\end{bmatrix} = \begin{bmatrix}
-\frac{\partial}{\partial t} & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
z_{10}(\zeta, t) \\
x_{10}(0, t)
\end{bmatrix} + \begin{bmatrix}
-1 \\
1
\end{bmatrix} x_{10}(0, t)
\]

\[
\dot{y}_{10}(t) = [C \quad B_{10}(L)] \begin{bmatrix}
z_{10}(\zeta, t) \\
x_{10}(0, t)
\end{bmatrix}
\]

where \[x_{10}(0, t) = \frac{\partial x_{10}(0, t)}{\partial t}\] and the operator \[C f(\zeta) = \int_{0}^{L} f(\zeta) d(\zeta - L) d\zeta = f(L).\]

Applying the Laplace transform to the above system, one obtains:

\[
\begin{bmatrix}
z_{10}(\zeta, s) \\
x_{10}(0, s)
\end{bmatrix} = \begin{bmatrix}
R_{11} & 0 \\
0 & R_{22}
\end{bmatrix} \begin{bmatrix}
z_{10}(\zeta, 0) \\
x_{10}(0, 0)
\end{bmatrix}
\]

where \[R_{11} = \int_{0}^{s} e^{s} f(s) ds = \int_{0}^{s} e^{s} f(s) ds\] and \[R_{22} = \frac{1}{s}.\]

The discrete system can be expressed as:

\[
\begin{align*}
Z_{0}(k) &= A_{d}Z_{0}(k-1) + B_{d}U_{0}(k) + E_{d}G_{0}(k) \\
Y_{0}(k) &= C_{d}Z_{0}(k-1) + D_{d}U_{0}(k) + F_{d}G_{0}(k)
\end{align*}
\]

The discrete operators \[A_{d}, B_{d}, C_{d}, D_{d}, E_{d}, F_{d}\] can be directly obtained. The original state can be obtained by the transformation: \[x_{10}(\zeta, k) = z_{10}(\zeta, k) + B_{10}(\zeta)x_{10}(0, k).\]

In this section, the discrete state space settings of the solar thermal system, BTES system, heat exchanger system, STTS system, natural gas boiler system and the district heating loop system are obtained. In the next section, we design a controller which maintains the temperature at desired set point, while still fulfilling the energy demands of the district heating loop.

### 4. Controller design and system analysis

Since large disturbances from the solar thermal plant system, borehole thermal energy storage system or the district heating loop system greatly impact system operation, the control system plays an important role in maintaining the system’s performance. In this section, we propose a servo controller design which successfully rejects undesired disturbances and tracks a reference trajectory or a set point.

One of the important analysis tools of the controlled system performance is given by the frequency analysis. The core of the frequency analysis is the frequency response of the system. In particular, we obtain frequency responses of the subsystems described in the previous section. The frequency response of the subsystems and units provides an insight into operational and performance capabilities, and also provides information on disturbance influence on the overall system’s performance.

#### 4.1. Servo control for linear discrete system

The performance of the servo controller design requirement is to maintain desired temperature of the flow supplied to the district heating loop system and reject disturbances simultaneously. In this work, we consider that the system operates during the heating phase.
If the solar thermal system and the BTES can not provide enough thermal energy, a backup natural gas boiler system is provided to ensure the necessary supply of thermal energy. Therefore, the control strategy of the solar thermal system with borehole seasonal storage is realized by the servo controller design for the natural gas system, see Fig. 3.

In this section, we design a servo controller for the discrete natural gas system. According to Eq. (34), the transfer function of the discrete system can be expressed as follows:

$$G_P(z) = \frac{z + 1}{(z + \beta_2)z - (z - \beta_2)}$$

(41)

Here, we assume that disturbances from different systems (solar thermal system, heat exchanger system, BTES and hot tank system) are harmonic function which model various sources with different frequencies. The servo control problem design is to track step reference trajectory and reject harmonic disturbances with the frequencies $\omega_1$ and $\omega_2$. Therefore, the transfer function of the controller contains the family of poles of functions chosen to be tracked and rejected as disturbance signals, such that:

$$C(z) = \frac{a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5}{(z - 1)(z^2 - 2 \cos(\omega_1 h)z + 1)}$$

(42)

The variables $z$ and $s$ are related as $z = e^{s \Delta t}$ when the system is mapped from continuous time domain to discrete time domain. Using Cayley-Tustin time discretization, the difference approximation corresponds to the series expansion of $z = e^{s \Delta t} = \frac{1 + s \Delta t}{1 - s \Delta t}$ which yields the following expression:

$$s = \frac{2}{\Delta t} \frac{z - 1}{z + 1}$$

(45)

here $\Delta t$ is the sampling period. The discrete transfer function is obtained by replacing $s$ in Ref. $G_1(s)$ by the above equation.

The frequency response of the above solar thermal system is plotted in Bode diagram, see Fig. 5. In this Bode diagram, a comparison between continuous (solid line) and discrete frequency responses (dash line or dash-dot line) with different sampling times is given. The Nyquist frequency for two different discretization sampling times are given as $\omega_{n1} = 34.14 rad/s (\Delta t = 0.1)$ and $\omega_{n2} = 62.83 rad/s (\Delta t = 0.05)$. From the figures, it can be seen that the magnitude curves are very close for frequencies that are much smaller than the Nyquist frequency and the phase curves coincide. This is in agreement with physical plant features that high frequency signals will be attenuated in the solar thermal plant system. It is obvious that as the sampling time decreases, the magnitude curve is closer to the magnitude curve of the continuous system. In

**Remark 4.** When it comes to the realization of a discrete controller, one needs to be careful in designing digital discrete state space realization of elements formulated in the Cayley-Tustin discretization framework. In particular, the appropriate care is required for application of the algorithm to the nominal discrete plant in the real time control setting.

### 4.2. System analysis based on frequency response

The frequency response is based on the fact that a linear system can be completely characterized by its steady-state response to harmonic signals [17,18]. Therefore, we can extend these results to discrete infinite and coupled infinite and finite dimensional systems. Based on frequency response, performance requirements can be expressed and in addition the evaluation of the effects of noise in the system can be achieved. In this section, we will explore frequency responses of the discrete subsystems described above.

First, let us consider the frequency response of the solar thermal energy system. The continuous transfer function of the solar thermal energy system is obtained from Laplace transform as follows:

$$G_1(s) = \frac{Y_1(s)}{U_1(s)} = \frac{\beta_1 \gamma_1}{(s + \beta_1)(s + \beta_2)} \left[1 - e^{(s + \beta_1)}\right]$$

(44)

The variables $z$ and $s$ are related as $z = e^{s \Delta t}$ when the system is mapped from continuous time domain to discrete time domain. Using Cayley-Tustin time discretization, the difference approximation corresponds to the series expansion of $z = e^{s \Delta t} = \frac{1 + s \Delta t}{1 - s \Delta t}$ which yields the following expression:

$$s = \frac{2}{\Delta t} \frac{z - 1}{z + 1}$$

(45)

Please cite this article in press as: Q. Xu, S. Dubljevic, Modelling and control of solar thermal system with borehole seasonal storage, Renewable Energy (2016), http://dx.doi.org/10.1016/j.renene.2016.05.091

Fig. 6. Magnitudes of Bode diagrams for the discrete BTES system in Eq. (45) (G2), heat exchanger system in Eq. (47) (G3), hot tank system in Eq. (48)–(49) (G41 and G42) and natural gas system in Eq. (50) (G5). The y-axis on the left hand side is for G2, G3, G41 and G42 and the y-axis on the right hand side is for G5. The sampling period is $\Delta t = 0.1$. 

addition, if one would consider to apply an output feedback control realization by placing a local gain based controller, the gain margin of the solar thermal system is 35 dB.

The frequency response of the BTES system is similar in nature to the frequency response of the solar thermal system with different parameters. The continuous transfer function of the BTES system is given as follows:

$$G_2(s) = \frac{Y_3(s)}{U_2(s)} = \frac{\beta_3 \gamma_2}{(s + \beta_3)(s + \beta_4)} \left[ 1 - e^{(s+\beta_3)} \right]$$  \hfill (46)

The Bode plot of BTES system is given in Figs. 6–7.

For the heat exchanger system, the continuous transfer function relates output $\dot{x}_6(L, t)$ to the input $\dot{x}_8(L, t)$, and is given as follows:

$$G_3(s) = \frac{\dot{x}_6(L, s)}{\dot{x}_8(L, s)} = \frac{2\beta_6 \sinh(b)}{2b \cosh(b) + (2s + \beta_5 + \beta_6 \beta_6) \sinh(b)}$$  \hfill (47)

where $b = \sqrt{\frac{(\beta_5 - \beta_3^2)^2}{4} + s^2 + (\beta_5 + \beta_6)s}$. One can directly obtain the discrete transfer function and frequency response based on the above continuous transfer function of the system. The Bode plot is also given in Figs. 6–7.

For the natural gas system, the continuous transfer function from the heat exchanger system $\dot{x}_{in}(t)$ to $\gamma_4(t)$ is obtained as follows:

$$G_4(t) = \frac{\dot{y}_4(s)}{\dot{x}_{in}(s)} = \frac{\beta_7 u_{in}}{X_{75} \left( s + \frac{2}{\gamma_3} \right)}$$  \hfill (48)

Fig. 7. Phases of Bode diagrams for the discrete BTES system in Eq. (46) (G2), heat exchanger system in Eq. (47) (G3), hot tank system in Eq. (48)–(49) (G41 and G42) and natural gas system in Eq. (50) (G5). The y-axis on the left hand side is for G2, G3, G41 and G42 and the y-axis on the right hand side is for G5. The sampling period is $\Delta t = 0.1$.

Fig. 8. Simulation of the solar thermal system profile given by the implementation of the discrete system in Eq. (14). The parameters of the system are $a_1 = 1$, $\beta_1 = 0.3$, $\beta_2 = 0.4$ and $\gamma_1 = 0.4$. The input $u_1(t)$ is the periodic harmonic function containing two frequencies $\omega_1 = 0.3142$ and $\omega_2 = 0.4064$.

Fig. 9. Output profile of simulation of the solar thermal system given by the implementation of discrete system in Eq. (14).

Fig. 10. Simulation of the natural gas system profile given by the implementation of the discrete system in Eq. (34). The parameters of the system are $\beta_0 = 1$ and $\gamma_6 = 1.5$. The input $u_6(t)$ is obtained by the servo controller in Eq. (42).
Similarly, the continuous transfer function from $\dot{x}_{8in}(t)$ in the BTES system to $\dot{y}_{4}(t)$ is obtained as follows:

$$G_{42}(s) = \frac{\dot{y}_{4}(s)}{\dot{x}_{8in}(s)} = -\frac{\beta_{7}u_{3SS}}{\gamma_{7}x_{3SS} + s + \frac{\beta_{7}}{\gamma_{7}}}$$ (49)

The discrete transfer functions and frequency responses are directly obtained based on the above continuous transfer functions of the system. The Bode plots of $G_{41}$ and $G_{42}$ are given in Figs. 6–7.

For the natural gas system, the open loop system discrete
transfer function is described in Eq. (41) and the discrete transfer function of the controller is described in Eq. (42). Thus, the discrete transfer function of the close loop system from $x_{in}(k)$ to $y_{c}(k)$ is expressed as follows:

$$G_2(z) = \frac{\tilde{y}_2(z)}{x_{in}(z)} = \frac{\tilde{\beta}_2 G_p(z)}{1 + \gamma_6 G_p(z) C(z)}$$  (50)

The discrete frequency response is obtained from the above discrete close-loop system. The Bode plot is given in Figs. 6–7.

Finally, the transfer function of the district heating loop system from $x_{10}(0, t)$ to $\tilde{y}_6(t)$ is obtained as follows:

$$G_6(s) = \frac{\tilde{y}_6(s)}{x_{10}(0, s)} = e^{-s}$$  (51)

This district heating loop system is a pure time delay system. The magnitude of the system is 0 and the phase of the system is $-\pi$.

The frequency response of the above subsystems are plotted in Bode diagram, see Figs. 6 and 7. The sampling period is $\Delta t = 0.1$. In the natural gas system, we assume the disturbances are with frequencies of $\omega_1 = 0.3142$ and $\omega_2 = 0.2199$. These frequencies can be reflected in the Bode diagram of the transfer function $G_5$.

5. Simulation results

In this section, we demonstrate the implementation of the servo control system to improve the overall efficiency of the system. The dynamic model of the collection-storage-district heating loop system is simulated according to the energy balance models developed in the previous section. With plant model available, a servo problem is set up to compute the control input that maintains the energy demand constant and rejects disturbances, with guaranteed asymptotic stabilization despite uncertainties present within the system.

In the next, we introduce two simulation scenarios. First, the servo control problem rejects disturbances arising in the solar thermal system. In the second scenario, the disturbances are arising from operating conditions of the district heating loop system and the BTES system.

5.1. Cloudy day: disturbances from the solar thermal system

We consider a scenario when a cloudy day with larger variations of available solar energy, the solar thermal system undergoes disturbance in the power output. Due to the weather changes and according to the weather forecast, the possible disturbances to the solar thermal system with the borehole seasonal storage can be considered as periodic harmonic disturbances with different frequencies. The control goal is to maintain the temperature of hot flow to the district heating loop system at desired set point and to reject two disturbances described above. The simulation results show 4 days (96 h) operation of the solar thermal system with $d_{c} = 0.01$ and $dt = 0.1$, see Figs. 8–12. The initial conditions of all states are zeros.

The solar thermal system is simulated as exposed to periodic harmonic disturbances given by two frequencies $\omega_1 = 0.3142$ and $\omega_2 = 0.4084$, see Figs. 8 and 9, where the input and output of the solar thermal system are given. Fig. 11 shows the simulation result of the heat exchanger’s two states. The natural gas system with servo control is given in Fig. 10. The desired poles of the designed controller are $s_{ACL} = \{0.65, 0.65, -0.75, -0.75, -0.55, 0.85\}$. It can be seen that the system can track the step reference $y_r = 1$ and reject periodic harmonic disturbances with different frequencies. The designed controller has good performance since it can achieve tracking the step reference in less than 5 h. However, one can easily reconfigure the controller and have faster tracking by placing $s_{ACL}$ closer to the center within the unit circle. Finally, Fig. 12 shows the simulation result of the district heating loop system. From the simulation result, it is obvious that the district heating loop system is driven by the input from the natural gas system.

5.2. Disturbances from operating conditions of the district heating loop system

When the operating conditions of the district heating loop system are affected by the environment changes, the heating loop system undergoes disturbance in power output. When the BTES system undergoes disturbance from the perturbations of the environmental temperature, these two disturbances will influence the solar thermal system with borehole seasonal storage. In this scenario, the control goal of the controller design is similar as the previous scenario. Therefore, the controller rejects disturbances in the district heating loop system and the BTES system and maintains the required flow temperature into the homes in the district heating loop system. The simulation results are shown in Figs. 13–17. The initial conditions of all states are zeros.

The BTES system is simulated with harmonic disturbance in the frequency of $\omega_1 = 0.2199$, see Fig. 13. Fig. 14 gives the input and
output of the BTES system. Here, the harmonic disturbance with the frequency of $\omega = 0.3142\pi$ is also considered as in-domain input to the heating loop system, see Fig. 16. The natural gas system with servo control is shown in Fig. 15. The desired poles of the designed controller are $\sigma(A_G) = \{0.5, 0.5, -0.8, -0.8, -0.7, 0.6\}$. As it can be seen from the simulation result, the designed controller has good tracking and rejecting performance. Finally, Fig. 17 shows the simulation result of the district heating loop system. This second scenario study shows that the designed controller has the ability to reject any linear combination of signals with known frequencies. In addition, the designed controller is easily realized in practice to address a wide range of disturbances.

6. Conclusion

In this work, we provided a model of the state-of-the-art in the solar thermal system with borehole seasonal storage mathematically modelled by ordinary differential equations (ODEs), hyperbolic partial differential equation (PDEs) and coupled PDEs-ODEs according to the energy balance. Then, the discrete systems of these integrated systems are obtained by the application of the Cayley-Tustin time discretization method. We developed a simple servo controller design for the solar thermal system which takes into account measurements of the disturbances. The control system manipulates the natural gas energy into the system in order to track a step reference for fulfilling the demands of space heating in the district heating loop system. The simulation results of different scenarios show that, the discrete servo controller tracks step reference and rejects harmonic disturbances with different frequencies. More advanced control and optimization schemes can be pursued in order to leverage the thermal energy storage. It is recommended that optimal control schemes are developed to help the solar thermal system with borehole seasonal storage to operate more efficiently.

Appendix. Resolvent of the heat exchanger system

The resolvent of the operator $A_{22}$ in Eq. (25) can be expressed as follows:

$$
R_{21} = \frac{b e^{a(1-\eta)} \sin h(b(1-\eta))}{e^{a(1-\eta)} \cos h(b(1-\eta))}
+ \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta
- \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta
- \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta

R_{22} = \frac{b e^{a(1-\eta)} \sin h(b(1-\eta))}{e^{a(1-\eta)} \cos h(b(1-\eta))}
+ \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta
- \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta

R_{23} = \frac{b e^{a(1-\eta)} \sin h(b(1-\eta))}{e^{a(1-\eta)} \cos h(b(1-\eta))}
+ \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta
- \int_0^1 [e^{a(1-\eta)} \cos h(b(1-\eta))] (-\eta) \, d\eta

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\[ R_{24} = \frac{\partial_p \alpha_c^C}{\partial f} \sin h(b \zeta) \left[ e^{b(1-\eta)} \cos h(b(1-\eta)) \right] \]
\[ + c e^{b(1-\eta)} \sin h(b(1-\eta)) \int_0^1 \left[ e^{\phi(1-\eta)} \cos h(b(1-\eta)) \right] \beta_5 B_6(\eta) \frac{1}{S} \cdot d\eta \]
\[ - \frac{\zeta}{b} e^{a(\zeta-\eta)} \sin h(b(\zeta-\eta)) \beta_5 B_6(\eta) \frac{1}{S} \cdot d\eta \]
\[ R_{33} = \frac{1}{S} \cdot (\cdot) \]
\[ R_{44} = \frac{1}{S} \cdot (\cdot) \]

where \( a = \frac{\partial_c - \beta_c}{2}, b = \sqrt{\frac{\partial_c - \beta_c}{4} + s^2 + (\beta_5 + \beta_6)s} \) and \( c = \frac{2s + \partial_c}{2b} \).

References


